Modified Newtonian Dynamics

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Abstract

This thesis will begin with the disscussion of the basic theory of Modified Newtonian Dynamics. It will then be concerned with questions of energy conservation, which does not seem to hold with the basic MOND principle. Also a few 'small' scale phenoma will be regarded, such as the Pioneer anomaly. The thesis will end with a discussion of MOND opposed to Dark matter.

Contents

1 Introduction

1.1 The problem

In the mid twentieth century observations of galaxies and galaxy clusters were made to determine there masses. The masses of these clusters were estimated based on its motion near the outer edges and using the virial theorem. These observations were then compared with the mass estimated using the total brightness of the cluster.
Both estimates strongly disagreed with each other. Up to a few hundred times more mass was estimated using the virial theorem then the mass calculated with the brightness of the cluster.
Astronomers were now presented with a new problem. The motion of galaxies could not be explained with the mass that was observed. This meant the either our basic principles of physics were incomplete, or that there was something which escaped the astronomers eye.

1.2 Dark matter

The first and most popular explanation that was offered to explain the apparent mass discrepancies, was to assume that there exists some form of 'dark matter'. This matter has the property that it interacts as normal matter in the gravitational sense, but that is does not interact with electro-magnetic radiation, making this matter undetectable.
In addition to the dark matter supposition, there is the assumption of some form of dark energy. Dark energy is supposed to permeate the entire universe and can explain the accelerating rate at which the universe is expanding.
According to the dark matter and energy theorem roughly 70% of the universe is dark energy, 25% is dark matter and only 5% of the univserse is made out of visible matter.

1.3 MOND

In a series of three papers [1], [2] and [3], Mordehai Milgrom proposes an alternative to the dark matter theorem. Rather than to assume that there exists some form of unseen mass or energy, Milgrom asserts that a change in the laws of dynamics can also explain observations. Milgrom proposes to change the laws of dynamics in such a way that, in the limit of very small accelerations, the force on a body is not proportional to the acceleration anymore. Instead in this limit the force becomes quadratically proportional to the acceleration. This assumption would explain the flattening of the rotational velocity curve of bodies far away from the center of the galaxy. His theory has become known as 'Modified Newtonian Dynamics' (MOND).

1.4 Contents of this thesis

A couple of things about Modified Newtonian dynamics will be looked at in this thesis. First the basic idea's of MOND will be looked at. This is done in section two, where the basic change in Newton's second law will be discussed. Secondly we will look at the consequences for galactic systems, and specifically

the rotational velocity curves of galaxies, the main phenomenon the MOND theory is designed for to explain. Also the new constant of nature a_0, which has the dimensions of acceleration. Specifically we are going the look at different methods of Milgrom to determine a_0, and see at what values he arrives. Also in section two, the discussion will be about the interpretation of basic MOND change. Should this change be regarded as a change of Newton's law of gravity, or is the change to be seen as a change of inertia?
In section three questions of energy will be regarded. Namely we see what happens to the kinetic energy of a particle when accelerated in the MOND regime. Also we want to check if conservation of energy still applies, assuming the original MOND idea holds.
In section four a phew phenomena of non-galactic scale will be considered. We see what kind of consequences there are if the constant a_0 is changed is such a way that the MOND regime applies for the case considered. A brief analysis of ocean tides will be given and we will also look at the pioneer anomaly.
Finally in section five we will look at some further developments of MOND which have been made since the theory first saw the light.
The thesis will end with a few words of criticism towards MOND.

2 The basics of MOND

2.1 Basic theory

Of course everyone is familiar with Newton's second law of dynamics:

$$\vec{F} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}, \tag{2.1}$$

which tells us that the acceleration of a particle is directly proportional to its mass and to the force acting on the particle. In the theory of MOND this second law is slightly altered. Milgrom proposes to include a function $\mu(x)$ into the second law with the stipulation that:

$$\mu(x \ll 1) = x \quad \mu(x \gg 1) = 1. \tag{2.2}$$

The modified form of the second law would then take the following form:

$$\vec{F} = m\mu(x)\vec{a}, \tag{2.3}$$

where

$$x = \frac{|\vec{a}|}{a_0}. \tag{2.4}$$

In this equation a new constant of nature a_0 is introduced, which has the dimensions of acceleration and is defined in such a way that we get the normal form of the second law for $|\vec{a}| \to \infty$, and for $|\vec{a}| \to 0$ we are in the so called DEEP MOND regime where:

$$\vec{F} = m\frac{|\vec{a}|}{a_0}\vec{a}. \tag{2.5}$$

2.2 Consequences for gravitational systems

The flattening of the rotational velocity curves near the outer regions of galaxies can easily be explained by MOND. If Newton's law of gravitation:

$$F = G\frac{mM}{r^2}, \tag{2.6}$$

is combined with the centripetal force law:

$$a = \frac{v^2}{r}, \tag{2.7}$$

then one can obtain the following relation for the velocity distribution for very large radii:

$$\mu\left(\frac{v^2}{ra_0}\right)\frac{v^2}{r} = \frac{GM}{r^2} \Longrightarrow v(r \to \infty) \approx \sqrt[4]{GMa_0}, \tag{2.8}$$

where M is mass of the bodies being orbited (in this case the total mass of the galaxy). This means that for $r \to \infty$ the orbital velocity of a body becomes independent of the orbital radius and tends to a constant value of $\sqrt[4]{MGa_0}$. The precise form of the function $\mu(x)$ is unknown but can be chosen such that it fits observed velocity curves.

Although most of the MOND theory is build upon equation (2.3), Milgrom stresses in [1] that this can only be considered a working formula, and that there is still a need for an underlying physical principle from which this equation can be derived. Nevertheless a lot of observations can be explained with formula (2.3).

2.3 Determination of a_0

Milgrom uses a number of methods in order to determine the new constant a_0. These methods will now be briefly looked at.

2.3.1 Fitting velocity curves

The most straightforward method to determine the constant a_0 is to calculate theoretical velocity distribution curves and then choosing a_0 such that is fits the observed curves. Milgrom does this in [2] where he makes use of a function[1]:

$$\xi \equiv \sqrt{\frac{MG}{a_0 h^2}} = \frac{v(r \to \infty)^2}{a_0 h} = \frac{v_\infty^2}{a_0 h}. \tag{2.9}$$

In this equation v_∞ is the orbital velocity of bodies with very large orbital radius, and h is a characteristic galaxy length scale. Now define $s \equiv \frac{r}{h}$ and $V(s) \equiv \frac{v(r)}{v_\infty} = \frac{v(sh)}{v_\infty}$. The first part of equation (2.8) can now be rewritten in a dimensionless form:

$$\frac{V^2\xi}{s}\mu\left(\frac{V^2\xi}{s}\right) = \frac{\xi^2}{s^2}. \tag{2.10}$$

The parameter ξ is a measure of the average acceleration in the galaxy in terms of a_0. If the average acceleration is very small then we have that ξ becomes very small and in this limit $V(s, \xi \to 0) = 1$, so that the orbital velocity tends, as expected, to a constant value. Also if the orbital radius becomes large the same limit conditions apply so that finally:

$$V(s \to \infty, \xi) = 1 = V(s, \xi \to 0) \Longrightarrow V(s \to \infty, \xi \to 0) = 1. \tag{2.11}$$

Equation (2.10) defines a collection of velocity curves, depending on ξ. One additional requirement is necessary to make a plot of s versus V, and that is to specify the interpolating function $\mu(x)$. This is done in an arbitrary way. In Milgrom's derivation he uses $\mu(x) = x(1+x^2)^{1/2}$. There are of course differences in calculating velocity curves for different types of galaxies. To describe how these differences affect the determination of a_0 goes into to much detail here, but there is more on this in [2]. Now all the ingredients are there to make an estimate of a_0. This involves assuming values of ξ so that it fits observed velocity curves. Using this method Milgrom comes to the rough estimate of:

$$a_0 = \alpha(1,9 \pm 1,2) \times 10^{-10}\frac{m}{s^2}, \tag{2.12}$$

where $1 \leq \alpha \leq 4$ is a parameter depending on the type of galaxy.

2.3.2 Tully-Fischer

Another way in which a_0 can be obtained, is to make use of the Tully-Fischer relation[2]. This relation suggests a correspondence between the luminosity of

[1] For the complete derivation, see:
M. Milgrom, 1983, 'A modification of the Newtonian dynamics: implications for galaxies', The Astrophysical Journal **270**: 371-383, paragraph 2.

[2] For the complete derivation, see:
M. Milgrom, 1983, 'A modification of the Newtonian dynamics: implications for galaxies', The Astrophysical Journal **270**: 371-383, paragraph 3.

galaxies and the rotational velocity at some point in the velocity curve. The correspondence is of the form $L \propto v^{\beta}$, were β is found between 2,5 and 5. In this relation the velocity that should be used is $v_{\infty}^4 = a_0 GM$. Because the luminosity must be a measure of the mass, galaxies should be used with a constant $\frac{M}{L}$ ratio. Now because M is proportional to v_{∞}^4 (according to MOND) and $\frac{M}{L}$ is assumed constant, it must be that $\frac{v_{\infty}^4}{L}$ is proportional to $\frac{M}{L}$. Milgrom uses this in combination with existing data to come to an estimate for a_0 of:

$$a_0 = 1,9 \times 10^{-8} h_{50}^2 \frac{cm}{s^2}, \tag{2.13}$$

where:

$$h_{50} = \frac{H_0}{50kms^{-1}Mpc^{-1}}, \tag{2.14}$$

with an error of a factor 2 and where H_0 is the Hubble constant.

2.4 Modified inertia or gravity?

One point of discussion is whether the MOND change is a change of inertia or a change of purely gravitational systems. If the change is one that only has to deal with gravitational forces, then equation (2.3) can be rewritten as:

$$\vec{g} = a_0 I^{-1} \left(\frac{\vec{g}_N}{a_0} \right) \vec{e}_N. \tag{2.15}$$

In this equation $\vec{g}$ is the modified gravitational field. $\vec{g}_N$ is the conventional gravitational acceleration in the $\vec{e}_N$ direction. The inverse function I^{-1} comes from the function $I(x) = x\mu(x)$. In the modified gravity form only the gravitational forces have to be considered in the modified dynamics, whereas in the modified inertia all combinations of forces have to be considered (electro-magnetic forces, for example).

Although there not much differences between modified gravity and modified inertia at galaxy scale, there are a few differences worth mentioning[3]. One difference is when different forces (gravitational and non-gravitational) are present. In such a situation where, for example, a large gravitational acceleration is nearly canceled by an electric force, resulting in a small acceleration, there is a MOND difference with the modified inertia, while there isn't a modification with the modified gravity. Another difference is the way in which conserved quantities are defined. The actions from which these quantities are derived differ with the modified inertia clause, but not in the modified gravity case.

2.5 The constant a_0

A more accurate value of a_0 is:

$$a_0 \approx 1,2 \times 10^{-10} \frac{m}{s^2}, \tag{2.16}$$

found by Begeman, Broeils and Sanders (1991). It is not entirely clear what the significance of this new constant is. The question is whether this constant

[3]See:
M. Milgrom, 2005, 'MOND as modified inertia', EAS Publications Series.

follows from some more fundamental constant or principles of physics, or that the constant is itself fundamental. There are a few interesting facts about a_0, found out by Milgrom and others which will be listed here:

- If the speed of light is divided by the lifetime of the Universe one gets approximately a_0. This means that particles with acceleration a_0 will reach the speed of light in the lifetime of the universe.
- In equivalence with the Planck length and the Planck mass, which are constructed from the constants $\hbar, G$ and c, one can construct the length scale $l_0 \approx 10^{27} m$, and the Mass scale $M_0 \approx 6 \times 10^{23} M_{sun}$. These numbers should be interpreted as boundary's from which MOND effects are to be expected.
- $2\pi a_0 \approx cH_0$.

The possible significance of these facts is presently unknown. It could be that they suggest a connection between MOND and cosmological phenomena, but they may also very well be shear coincidences.

3 Energy

One question that naturally arises is whether conservation of energy is satisfied for Milgrom's modified form of dynamics. This issue will now be looked at, starting from equation (2.3).

3.1 Kinetic energy

If a particle is accelerated from a speed v_1 to a speed v_2, then in normal Newtonian dynamics the work done is well known:

$$W_{12} = \frac{1}{2}m(v_2^2 - v_1^2) = \Delta E_{kinetic}, \tag{3.1}$$

where m is the particle mass and $\frac{1}{2}mv^2$ is the kinetic energy of the particle. The work done is thus equal to the change in kinetic energy. The first question that must be asked here is if this law still applies if a particle is accelerated in the DEEP MOND[4] regime. In the DEEP MOND regime:

$$F = m\frac{a^2}{a_0}. \tag{3.2}$$

The work done in accelerating a particle from v_1 to v_2 with an acceleration $a \ll a_0$ can now be calculated:

$$\begin{aligned} W_{12} &= \int_1^2 F dr \\ &= \int_1^2 m\frac{a^2}{a_0} v dt \\ &= \frac{m}{a_0}\int_1^2 \left(\frac{dv}{dt}\right)^2 v dt \\ &= \frac{m}{a_0}\int_1^2 v' v dv \\ &= m\frac{a}{a_0}\left|\frac{1}{2}v^2\right|_{v_1}^{v_2} \\ &= \frac{a}{a_0}\left[\frac{1}{2}m(v_2^2 - v_1^2)\right] \\ &= \frac{a}{a_0}\Delta E_{kinetic}. \end{aligned} \tag{3.3}$$

It can now be seen that the work done in accelerating a particle is reduced by a factor $\frac{a}{a_0}$ in de DEEP MOND regime. This means that one consequence of the modified Newtonian dynamics is that the work done in changing a particles' speed, depends not only on the speed change, but also on the time it takes to change the speed. In the general case we have of course:

$$W = \mu\left(\frac{a}{a_0}\right)\Delta E_{kinetic} \tag{3.4}$$

In the next section we will see that this formula imposes restrictions on the function μ.

[4]DEEP MOND regime means we have accelerations for which holds: $a \ll a_0$.

3.2 Conservation of energy

One fundamental law that has to hold is conservation of energy. This means that in order for MOND to be consistent with the conservation of energy theorem, the interpolating function μ is most likely restricted in its form.
Imagine now the simple case where there are two particles, both with mass m, accelerating from zero speed to a certain speed v. Particle one is acted upon by a force F_1 which causes an acceleration $a_1 \gg a_0$. The second particle is acted upon by a force F_2 which causes an acceleration $a_2 \ll a_0$. When both particles arrive at speed v, they must both have the same kinetic energy. This means that the work done on the particles has to be the same. The first particle is accelerated in the Newtonian regime whilst the second particle is accelerated in the MOND regime. The work done on both particles is then:

$$\begin{aligned} W_1 &= F_1 x_1 = m a_1 \frac{1}{2}\frac{v^2}{a_1} = \frac{1}{2} m v^2 \\ W_2 &= F_2 x_2 = m \frac{a_2}{a_0} a_2 \frac{1}{2}\frac{v^2}{a_2} = \frac{1}{2} m v^2 \frac{a_2}{a_0}. \end{aligned} \tag{3.5}$$

Obviously there is a problem here. while both particles begin and end the equal speed, the work done in accelerating the particles to that speed is not the same. In the second case, where the acceleration is very low, the work done is reduced by a factor $\frac{a_2}{a_0}$. This was also calculated in the previous section. The strange thing is that the work done is *less* then the kinetic energy the particle is supposed to have. This means there is some energy missing.
In the general case where MOND dynamics is supposed to hold, when a set of particles, denoted by the index i, are accelerated to v, under the influence of a force F_i, then the work done on the particle i is equal to:

$$W_i = F_i x_i = m\mu\left(\frac{a_i}{a_0}\right) a_i \frac{1}{2}\frac{v^2}{a_i} = \mu\left(\frac{a_i}{a_0}\right)\frac{1}{2} m v^2. \tag{3.6}$$

Because every particle i traveling at speed v, has to have the same amount of energy E associated with it (this even doesn't necessarily have to be the normal kinetic energy $\frac{1}{2}mv^2$). To ensure conservation of energy it must be that:

$$W_i = E \quad \forall i. \tag{3.7}$$

This means that there are severe restrictions on the function μ:

$$\mu\left(\frac{a_i}{a_0}\right) = \mu\left(\frac{a_{i+1}}{a_0}\right) \quad \forall i. \tag{3.8}$$

This result means that the μ function is a constant function, which is in contradiction with the limit conditions that were imposed on μ in the first place. The conclusion which can be drawn from this result is that MOND in its original form (ie. as Milgrom proposed is in 1983) does not conserve energy. Less likely is that the work done on particles is also different when accelerated in the MOND regime. The analysis above would then be different, but it is assumed that the only change present is the MOND change.

4 Non-galactic scale phenomena

4.1 Earth's ocean tides

An interesting question is if the MOND effect can be measured with the earth's ocean tides. We will now discuss this issue. Doing this we will use a derivation which can be found in [4][5], and modifying the equations is such a way that MOND is incorporated into them. We will then look if the difference with normal Newtonian dynamics is large enough to be measured, and if it is not we will look at what happens when the parameter a_0 is altered. We will assume that the MOND change is one of inertia, so that Newton's law of gravitation applies:

$$\vec{F} = G\frac{mM}{R^2}\vec{e}_R. \tag{4.1}$$

If multiple gravitational forces act on a body with mass m we of course have that the resultant force is equal to:

$$\vec{F}_{resultant} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n = \sum_n \vec{F}_n. \tag{4.2}$$

This means that:

$$m\mu\left(\frac{a}{a_0}\right)a\vec{e}_R = m\sum_n \mu\left(\frac{a_n}{a_0}\right)a_n\vec{e}_{R_n}, \tag{4.3}$$

which in turn implies that:

$$\mu\left(\frac{a}{a_0}\right) = \mu\left(\frac{\sum a_n}{a_0}\right). \tag{4.4}$$

The derivation assumes a model in which the surface of the earth is completely covered with water, and will deal with only the moon and not the Sun. Earth is assumed to be perfectly spherical. Because the tides have to be described form an inertial frame of reference, an arbitrary point in space has to be chosen as the origin. Let the masses of the earth and the moon be M_{earth} and M_{moon} respectively. The radius of Earth is r and the distance from the center of Earth to the center of the moon is D. The effects of the Earth's and the Moon's gravitational pull can be investigated by imagining that a mass m is placed on an arbitrary point on the surface of Earth, and then looking at what the forces will do with this mass. The position vector of m as seen from the moon is $\vec{R}$ and is $\vec{r}$ as seen from the center of Earth. In the inertial frame of reference the system is described as follows: the position vector from the origin of the inertial frame of reference to the mass m on the surface of Earth is $\vec{r}_m$ and the position vector to the center of Earth is $\vec{r}_E$. The force $\vec{F}_{m-moon}$ on the mass m due to the gravitational pull from the moon is:

$$\vec{F}_{m-moon} = -G\frac{mM_{moon}}{r^2}\vec{e}_r. \tag{4.5}$$

[5]For the complete derivation, see:
S.T. Thornton & J.B. Marion, *Classical dynamics of particles and systems*, fifth edition, Belmont USA, Thomson - Brooks/Cole, 2004, paragraph 5.5, page 198-204

Similarly the force $\vec{F}_{m-earth}$ on the mass m is:

$$\vec{F}_{m-earth} = -G\frac{mM_{earth}}{R^2}\vec{e}_R. \tag{4.6}$$

The total force $\vec{F}_m$ on m is then:

$$\vec{F}_m = m\mu\left(\frac{\ddot{r}_m}{a_0}\right)\ddot{r}_m = -G\frac{mM_{moon}}{r^2}\vec{e}_r - G\frac{mM_{earth}}{R^2}\vec{e}_R. \tag{4.7}$$

Also, the force $\vec{F}_{ME}$ between the center of mass of the moon between the center of mass of Earth is equal to:

$$\vec{F}_{ME} = M_{earth}\mu\left(\frac{\ddot{r}_E}{a_0}\right)\ddot{r}_E = -G\frac{M_{earth}M_{moon}}{D^2}\vec{e}_D. \tag{4.8}$$

We want to describe the position of m with respect to the center of mass of Earth this means that we need to find $\ddot{r}$. This of course depends on the separate accelerations of m and the Earth's center of mass:

$$\begin{aligned}
\mu\left(\frac{\ddot{r}}{a_0}\right)\ddot{r} &= \mu\left(\frac{\ddot{r}_m}{a_0}\right)\ddot{r}_m - \mu\left(\frac{\ddot{r}_E}{a_0}\right)\ddot{r}_E \\
&= \frac{\vec{F}_m}{m} - \frac{\vec{F}_{ME}}{M_{earth}} \\
&= G\frac{M_{moon}}{r^2}\vec{e}_r - G\frac{M_{earth}}{R^2}\vec{e}_R + G\frac{M_{moon}}{D^2}\vec{e}_D \\
&= G\frac{M_{earth}}{R^2} - GM_{moon}\left(\frac{\vec{e}_R}{R^2} - \frac{\vec{e}_D}{D^2}\right).
\end{aligned} \tag{4.9}$$

This equation is to be compared with equation (5.50) in [4]. The only difference is the function μ in front of $\ddot{r}$. This could be expected because the terms on the right of the equal sign are the gravitational force contributions, and we had assumed the gravitational laws unaltered under MOND. In [4] a calculation is made to determine the maximum height change in the ocean tides (under the simplifying assumptions made). We will now attempt to follow the same method and determine whether there are any observable MOND effects. The second part of equation (4.9) is the part that is responsible for the ocean tides, so we have now that the tidal force $\vec{F}_{tidal}$ on a mass m on the surface of Earth is:

$$\vec{F}_{tidal} = -GmM_{moon}\left(\frac{\vec{e}_R}{R^2} - \frac{\vec{e}_D}{D^2}\right) \tag{4.10}$$

The tidal forces in the x and the y direction ($\vec{F}_x$ and $\vec{F}_y$) on an arbitrary point along the surface of Earth, as derived in [4], are:

$$F_x = 2G\frac{mM_{moon}x}{D^3}, \tag{4.11}$$

and

$$F_y = -G\frac{mM_{moon}y}{D^3} \tag{4.12}$$

In these equations $x = rcos(\theta)$ and $y = rsin(\theta)$, so we can now describe the tidal force for an arbitrary angle θ around Earth. If the tidal forces causes a mass m

to change in height by an amount h, then in Newtonian dynamics the difference in potential energy is just mg_Nh. In MOND dynamics this changes. In MOND dynamics one must use the modified gravitational field, given by equation (2.14), instead of the Newtonian gravitational acceleration g_N. In order to use equation (2.14), the function $\mu\left(\frac{a}{a_0}\right) = \mu\left(\frac{g}{a_0}\right)$ must be specified. In this derivation we use:

$$\mu(x) = \frac{x}{\sqrt{1+x^2}} = \frac{\frac{g}{a_0}}{\sqrt{1+\left(\frac{g}{a_0}\right)^2}} = \frac{g}{a_0\sqrt{1+\frac{g^2}{a_0^2}}}. \tag{4.13}$$

This equation will be used here because Milgrom uses it as an example in [2]. The form of this function assures that $g = g_N$ for $g \gg a_0$, and $g = \sqrt{a_0 g_N}$ for $g \ll a_0$. In [6], a full derivation is made of the modified gravitational field using (4.13). The derivation will not be followed here but is given to be:

$$g = a_0\sqrt{\frac{\left(\frac{g_N}{a_0}\right)^2 + \left(\frac{g_N}{a_0}\right)\sqrt{\left(\frac{g_N}{a_0}\right)^2+4}}{2}}. \tag{4.14}$$

To confirm this equation satisfies the limit conditions let $g_N \gg a_0$, then:

$$\lim_{g_N \gg a_0} g = a_0\sqrt{\frac{\left(\frac{g_N}{a_0}\right)^2 + \left(\frac{g_N}{a_0}\right)\sqrt{\left(\frac{g_N}{a_0}\right)^2}}{2}} = a_0\sqrt{\frac{2\left(\frac{g_N}{a_0}\right)^2}{2}} = g_N, \tag{4.15}$$

and the other limit $g_N \ll a_0$:

$$\lim_{g_N \ll a_0} g = a_0\sqrt{\frac{\left(\frac{g_N}{a_0}\right)\sqrt{4}}{2}} = \sqrt{a_0 g_N}. \tag{4.16}$$

The gravitational work W done in changing the tide height by h is calculated in [4] to be:

$$W = \frac{3GmM_{moon}r^2}{2D^3}. \tag{4.17}$$

In normal Newtonian dynamics this has to be equal to mg_Nh, and the maximum height change h is then calculated to be:

$$h = 0,54m. \tag{4.18}$$

In the modified dynamics the work is equal to mgh, where g is now given by equation (4.14), so the height is then inversely proportional to g, and the maximum height change is then:

$$h = 0.54m. \tag{4.19}$$

This result means that MOND effects cannot be seen using the tides on Earth. This was expected because we used $a_0 = 1,2 \times 10^{-10}$, and $g_N \gg a_0$ so that g is almost identical to g_N. Let us now change a_0 to $a_0 = g_N$. The maximum height change then becomes:

$$h = 0.42m. \tag{4.20}$$

Increasing a_0 causes the maximum tidal height to decrease. In the limit where the constant a_0 becomes very large, the ocean tides of the Earth vanish completely.

4.2 Planetary motions

The three laws of Kepler are:

1. *Planets describe elliptical orbits around the Sun, where the sun is at one of the foci.*
2. *The time derivative of the area swept out by a radius vector from the sun to a planet is constant.*
3. *The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of that planet's orbit.*

Kepler derived these laws empirically, but they can also be derived from Newton's laws of dynamics. An interesting question which then arises is whether Kepler's laws still hold with MOND or what kind of changes there are if MOND applies. The first attempt will be to see what happens to Kepler's second law when MOND is applied. If we imagine a two body system, where one body with mass m orbits the other with mass M in an elliptical orbit at a distance $\vec{r}(t) = r\vec{e}_r$. We have then:

$$m\mu\left(\frac{|\ddot{\vec{r}}|}{a_0}\right)\ddot{\vec{r}} = -G\frac{mM}{r^2}\vec{e}_r. \tag{4.21}$$

The acceleration of the body orbited can be neglected because we assume $M \gg m$. If $\vec{e}_\theta$ is the tangiental unit vector we have:

$$\dot{\vec{e}}_r = \dot{\theta}\vec{e}_\theta \quad \& \quad \dot{\vec{e}}_\theta = -\dot{\theta}\vec{e}_r. \tag{4.22}$$

The velocity and acceleration vectors can then be expressed as:

$$\dot{\vec{r}} = \dot{r}\vec{e}_r + r\dot{\vec{e}}_r = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta \tag{4.23}$$

$$\begin{aligned}\ddot{\vec{r}} &= \ddot{r}\vec{e}_r + \dot{r}\dot{\vec{e}}_r + \dot{r}\dot{\theta}\vec{e}_\theta + r\ddot{\theta}\vec{e}_\theta + r\dot{\theta}\dot{\vec{e}}_\theta \\ &= \ddot{r}\vec{e}_r + \dot{r}\dot{\theta}\vec{e}_\theta + \dot{r}\dot{\theta}\vec{e}_\theta + r\ddot{\theta}\vec{e}_\theta - r\dot{\theta}^2\vec{e}_r \\ &= \left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta. \end{aligned} \tag{4.24}$$

Equation (4.24) can now be plugged into equation (4.21) to obtain:

$$\begin{aligned}-G\frac{mM}{r^2}\vec{e}_r &= m\mu\left(\frac{\ddot{r}}{a_0}\right)\left(\left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta\right) \\ -G\frac{M}{r^2}\vec{e}_r &= \mu\left(\frac{\ddot{r}}{a_0}\right)\left(\left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta\right)\end{aligned} \tag{4.25}$$

Unfortunately these equations are very difficult, if not impossible to solve if the precise form of $\mu(x)$ is not known. However one could argue that Kepler's second law of planetary motion still holds, because this law also follows from the law of conservation of angular momentum, which is unchanged in Modified Newtonian dynamics. Concerning the third law it could very well be that this law of Kepler *is* changed in MOND. One could imagine for instance a planet orbiting a star in a very large elliptical orbit in Newtonian mechanics, which is distorted by

MOND effects if the planet is furthest away from that star. A certain points on the orbit MOND effects would rule, whereas on other points, the Newtonian rules are restored. Similarly Kepler's third law would also be altered, since the period of the orbit would also be altered in MOND when considering extreme large orbits.

4.3 The pioneer anomaly

One anomalous phenomenon in our solar system, for which the MOND theory can be a possible explanation, is the so called Pioneer anomaly. The Pioneer spacecrafts had as a primary mission to explore the outer planets of our solar system. Pioneer 10 was launched on March 2, 1973. The launch of Pioneer 11 took place on April 5, 1973. The Pioneer 11 spacecraft stopped transmitting on Oktober 1, 1990. At that time its distance from the Sun was build op to 30 AU. However the Pioneer 10 spacecraft was still transmitting until fairly recently. Its mission officially ended on March 31, 1997, at a distance of 67 AU from the Sun. Measurements on the acceleration of these spacecrafts can be done very precise. This is because the spacecrafts are spin-stabilized, and have to do a minimum number of correction maneuvers. The anomaly lies in the fact that the Pioneer 10 spacecraft is not where it is supposed to be, on the basis of calculations which have been made. The spacecraft is somewhat closer to the Sun than it is supposed to be. There appears to be an unexplained acceleration working on Pioneer 10, directed towards the Sun. This acceleration has a value of approximately:

$$a_{anomalous} \approx 8 \times 10^{-10} \frac{m}{s^2}. \tag{4.26}$$

One notices immediately that this value is of the same order as a_0. The significance of this fact is probably not very large, given the fact that is does not follow from the MOND equation that if one has accelerations in the non-MOND regime, the constant a_0 has to bee added or subtracted from the acceleration. The constant a_0 is only meant to serve as a comparative constant to determine whether one is in the MOND regime or in the Newtonian regime.
Although there is still no universally excepted explanation for the deviation, many theories can explain it to a certain degree of accuracy. Examples of this include external gravitational forces, malfunctions in the spacecrafts such as gas leaks, or errors in certain values such as the masses of planets. In this case an explanation using Modified Newtonian Dynamics is of special interest.
Because the pioneer spacecraft is still in the solar system, it is true that gravitational accelerations are still very large compared to a_0. It is then true that $\mu \approx 1$, so we can try to model the anomalous acceleration as a small perturbation of this μ:

$$\begin{aligned} -G\frac{M_{sun}}{r^2} = \mu\left(\frac{\ddot{r}}{a_0}\right)\ddot{r} &= (1-\epsilon)\ddot{r} \\ \ddot{r} &= -\frac{G}{1-\epsilon}\frac{M_{sun}}{r^2}. \end{aligned} \tag{4.27}$$

The effect of this would indeed be an increment in acceleration towards the sun. Doing it in this way would mean a modification of gravity instead of inertia, because what is effectively done is modifying the gravitational constant slightly. The acceleration towards the Sun is assumed to be constant, but within the error margins it could be that it also depends on the distance from the sun, as is expected using MOND. The error acceleration a towards the Sun should be equal to (if we assume a linear trajectory form the Sun for the sake of simplicity):

$$a = a_{detected} - g_N = \frac{G}{1-\epsilon}\frac{M}{r^2} - G\frac{M}{r^2}. \tag{4.28}$$

This equation can be rewritten in order to obtain ϵ:

$$\epsilon = \frac{ar^2}{ar^2 + GM}. \tag{4.29}$$

For this equation to be valid we need have small ϵ, so we can see that $ar^2 \ll GM$. If in equation (4.29) the found value of a is filled in and we use $r \approx 70AU$, we get:

$$\epsilon \approx 6,6 \times 10^{-10} \tag{4.30}$$

As seen from this value of ϵ, the deviation is indeed very small. If it is assumed the anomalous acceleration is purely caused by MOND deviations, then the result could be used to put constraints on the function μ. Because we now have:

$$\mu\left(\frac{\ddot{r}}{a_0}\right) = 1 - \epsilon. \tag{4.31}$$

Since one can measure $\ddot{r}$ and calculate ϵ, one knows the values of μ for different values of $\ddot{r}$. If these values are plotted, one gets a curve of μ. It is however more likely that the anomalous acceleration is caused by other effects. Namely because the anomaly is so large, that there is a measurable difference in the spacecrafts trajectory in comparison with the calculated trajectory. It should then also be possible to detect differences in planets orbits, but no observable differences are found there.

5 Further developments of the original MOND theory

5.1 MOND field equation

The basic MOND proposal as shown in section one, is effectively only a working equation. A better description for a MOND change can be sought in changing the gravitational potential, which is derivable from a lagrangian. The advantage of this approach is that, since the dynamics are then derivable from a langrangian, automatically all questions of conserved quantities are ruled out. A few examples of such equations will be given here, but for the full treatments the reader is recommended to read: [14]. In Newtonian dynamics, the acceleration $\vec{g}_N$ of a test particle in a gravitational field is given by:

$$\vec{g}_N = -\vec{\nabla}\phi_N. \tag{5.1}$$

If the particle is in a space with mass distribution ρ, then the gravitational potential is given by:

$$\vec{\nabla}^2\phi_N = 4\pi G\rho \tag{5.2}$$

This is the well known Poisson equation, which is derivable from the langrangian:

$$L_{Newton} = -\int \left(\rho\phi_N + \frac{\left(\vec{\nabla}\phi_N\right)^2}{8\pi G}\right) d^3r \tag{5.3}$$

An example of a MOND change in this lagrangian is to change ϕ_N to ϕ, arbitrary, and to change $\left(\vec{\nabla}\phi_N\right)^2$ to:

$$\left(\vec{\nabla}\phi_N\right)^2 \longrightarrow a_0^2\chi\left(\frac{\left(\vec{\nabla}\phi\right)^2}{a_0^2}\right), \tag{5.4}$$

where $\chi(x^2)$ is an arbitrary function just like $\mu(x)$. With this langrangian, the potential field equation becomes:

$$\vec{\nabla}\cdot\left(\mu\left(\frac{|\vec{\nabla}\phi|}{a_0}\right)\vec{\nabla}\phi\right) = 4\pi G\rho, \tag{5.5}$$

which is equivalent to:

$$\vec{\nabla}\cdot\left(\mu\left(\frac{g}{a_0}\right)\vec{g}\right) = -4\pi G\rho. \tag{5.6}$$

Here μ has the same limit properties as throughout this text, so that the normal Poisson equation is valid for $g \gg a_0$. As mentioned before the main advantage of this theory is that is conserves all quantities that must be conserved. The main disadvantage is that its an insolvable equation for most configurations of ρ.

5.2 Relativistic MOND

For a complete MOND theory, a relativistic adaptation of MOND is important. Moreover because there are also mass discrepancies with relativistic phenomena. There are a good number of theories devised for relativistic MOND, but the reader is referred to [8] for the most recent theory of J.D. Bekenstein, caller TeVeS (Tensor Vector Scalar). The derivation will not be shown in this thesis because it is to long and is not particularly enlightening for our purposes.

6 Criticism

6.1 MOND versus Dark matter

The main objection towards Modified Newtonian Dynamics is that the theory describes some phenomena so well because it is designed to do so. The theory is thus an effective theory designed specifically to fit the data. MOND has not been derived from any fundamental rules of physics. The MOND equation thus hasn't got any fundamental physical justifications but it a very ad hoc theory. Ideally physicists would like to have some new new fundamental principle of nature from which the equation $F = m\mu\left(\frac{a}{a_0}\right)a$ naturally follows. So Milgrom's method of finding his equation is somewhat outside the modern day physics methodology, which assumes some fundamental principles of nature are true, such as conservation of energy, and then deriving new equations and laws from those principles. Opposes to this argument one could argue that Newton's formula $F = ma$, is in fact also a effective formula. Newton did not derive the law from any fundamental principles. Indeed when people began to research very high speed's the law had to be altered. So while the MOND theory is an effective theory, one could argue that Newton's second law is also empirically established, so there is effectively no difference in how those laws are established. The way for the MOND theory to be discarded, is that direct observations are made of the missing mass in the universe, so that there is no need for MOND anymore. On the other hand there seems to be no easy and direct way in which MOND can be confirmed. The fact that the MOND theory itself is incomplete further complicates this matter. For instance it is still to be determined whether MOND is a modification of gravity or a modification of inertia. If it turns out to be a modification of gravity then it would be very hard to do experiments on Earth and even in the Solar system, because the sun makes sure we are always in a gravitational field $g \gg a_0$. If MOND turns out to be a modification of inertia, then experiments can be done with other forces, to detect MOND deviations. It would then be feasible to do experiments on earth, letting particles move under the influence of different force fields, such that a small net force remains, and then examining the particles' trajectories. The best way of making MOND more plausible is still to derive it from some new fundamental principles of physics.
One issue that also has to be considered the simplicity of the Dark matter theorem versus the MOND theorem. There is a theory which tells us that whenever there are two ore more theories describing the same phenomenon, the simplest one of the theories is usually the one that is true or has to be used. If one of the two theories has to be chosen using this criterium, it is still not clear whether one should go with Dark matter or one should go with MOND. Of course there is a fierce discussion between MOND fans and Dark matter fans, on which theory is the simplest. As there is still no data found which satisfactory discards MOND or Dark matter, some data favors MOND while other data favors Dark matter. So both MOND and Dark matter continuously have to be adapted to explain or fit new data. Fans of the Dark matter theorem claim it is simpler to assume the existence of dark matter, rather than to make changes to fundamental laws of physics. On the other hand fans of MOND argue that MOND is only a change of one law with the introduction of a single new constant, whereas there are several different dark matter theorems for different types of galaxies, with multiple variables needed.

The theory that shows the most promise at this moment is the TeVeS, relativistic theory of MOND. This theory has mainly been somewhat succesfull in explaining gravitational lensing. However to explain these new phenomena more and more new features are added to MOND, which continuously make it more complex and harder to understand. The debate on MOND versus Dark matter is a fierce one as can be seen, and is most likely to rage on until one theory has been satisfactory established or disproved.

6.2 My own opinion

While both theories can explain certain phenomena in the galaxies, i find it hard to believe that Dark matter or MOND is the answer to the problem. My main objection to MOND is that, as said in the previous section, the theory does not follow from some basic principles. Since the original MOND idea several adaptations have been made, each introducing new assumptions or formula's which can all describe the phenomena. I do not think physicists are there to make better formula's to describe certain processes, but they must search deep principles which can explain the entire universe. In the beginning equations are useful, but the real effort must be to explain the equations. I find Dark matter even harder to believe, because in my opinion it is a very easy solution, to claim that something exists which we cannot see. Since we cannot see it, it is also impossible to disprove it. If find it more likely to assume the laws of physics are incomplete than to assume the universe mass' is incomplete.

7 Final words

In this thesis we have looked at some basic principles of the Modified Newtonian Dynamics. Milgrom proposed this theory in 1983, and since than the theory has been developed further mainly by Milgrom and Bekenstein. Since the original MOND equation has several shortcomings, namely that is doesn't conserve energy and that the equation is very ad-hoc, all sorts of improvements have been made, to make it more plausible. MOND explains with success the flattening of galaxy velocity rotation curves, but the theory is not yet been sufficiently tested on smaller scale. In the future, when measurement systems become more accurate, test can be devised to detect very small MOND deviations at small scale, so that the theory can be dismantled or confirmed. Until that time, Dark matter remains the most popular theory opposing MOND.

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