Majorana Neutrinos and Neutrinoless Double Beta Decay

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Abstract

This bachelor thesis discusses the Majorana description of neutrinos. In this description, of neutrinos beyond the standard model, the neutrino is its own antiparticle. Mass terms involving these particles violate lepton number conservation. These facts make this description a candidate to describe neutrinoless double beta decay. Furthermore the see-saw mechanism and other mass models for neutrinos are discussed. Also the claim of the first measurement of neutrinoless double beta decay by [3] is reviewed. The thesis concludes that the see-saw mechanism is the best explanation for the neutrino masses and that detection of neutrinoless double beta decay would imply lepton number violation and massive neutrinos.

Contents

1 Introduction

1.1 Double beta decay

In 1987 the process of double beta decay was observed for the first time. In the experiment of Elliot, Hahn and Moe [1] a Selenium nucleus decays to a Krypton nucleus through the simultaneous emission of two electrons and two electron antineutrinos.

$$^{82}Se \rightarrow^{82} Kr + 2e^- + 2\bar{\nu}_e \tag{1.1}$$

In general such a process is described by the following decay equation and the Feynman diagram in Figure 1$^{a)}$.

$$N_{A,Z} \rightarrow N_{A,Z+2} + 2e^- + 2\bar{\nu}_e \tag{1.2}$$

This process will be indicated with $\beta\beta_{2\nu}$ decay from now on. $\beta\beta_{2\nu}$ decay can only occur if single beta decay of the initial nucleus is forbidden. This and the fact that is a second order weak process, makes $\beta\beta_{2\nu}$ decay a very rare process. Since 1987 a reasonable number of nuclei were observed to $\beta\beta_{2\nu}$ decay, an overview of these decays is given in table 14.1 of reference [2]. The results, with an average lifetime of 10^{20} years, indeed show that the process of double beta decay is very rare.

1.2 Neutrinoless double beta decay

The process of neutrinoless double beta decay, denoted by $\beta\beta_{0\nu}$, is almost similar to the $\beta\beta_{2\nu}$ process. It is described by the following decay equation and the Feynman diagram in Figure 1$^{b)}$.

$$N_{A,Z} \rightarrow N_{A,Z+2} + 2e^- \tag{1.3}$$

From this equation it is seen that $\beta\beta_{0\nu}$ decay violates lepton number conservation. Thus in contrast to the $\beta\beta_{2\nu}$ decay, the $\beta\beta_{0\nu}$ decay cannot occur within the framework of the Standard Model. In 2002 the Heidelberg-Moscow collaboration claimed to have observed neutrinoless double beta decay [3]. In their

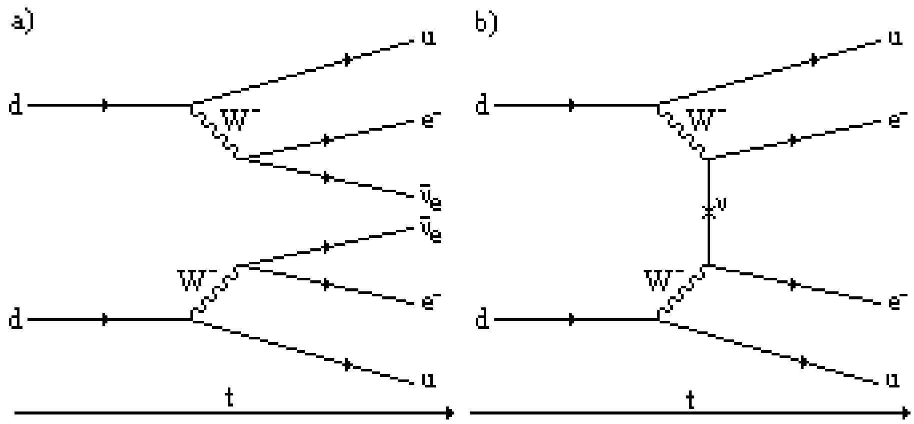

Figure 1: Feynman diagram of $\beta\beta_{2\nu}$ decay $^{a)}$ and $\beta\beta_{0\nu}$ decay $^{b)}$

article they present a statistical analysis of ten years experiment on the double beta decay of ^{76}Ge. This claim has been criticized by numerous physicist in similar research areas. This thesis will discuss a theoretical description beyond the Standard Model, which makes $\beta\beta_{0\nu}$ decay possible. This is the 'Majorana' description of neutrinos.

1.3 Majorana neutrinos

The defining property of a Majorana neutrino is that it is its own antiparticle. These and other properties of the Majorana neutrino need to be incorporated in the description of double beta decay. For the description of double beta decay two theories are important. First the description of free spin $\frac{1}{2}$ particles: the Dirac equation, and second the charged current weak interactions in the Standard Model. This is the interaction that describes the decay of a neutron (udd) to a proton (uud), an electron and an electron-neutrino, which occurs in beta decay processes:

$$n \rightarrow p + e^- + \nu_e \quad \text{or} \quad u \rightarrow d + e^- + \nu_e \tag{1.4}$$

In section two of this thesis properties of Majorana neutrinos in the Dirac equation are discussed. Topics of interest are the structure of the Majorana spinor, Majorana mass terms in the Lagrangian and the mass and charge of Majorana particles. The charged current weak interaction and properties of the neutrino in the Standard Model are discussed in section three. In section four the requirements for $\beta\beta_{0\nu}$ decay are discussed and the topic of section five is the experiment itself. Note that references [1] to [6] are explicitly used in this thesis, reference [7] and [8] are general texts on relativistic quantum mechanics and references [9] to [16] are interesting articles on neutrinoless double beta decay.

2 Properties of the Dirac equation

In this thesis natural units will be used, i.e. $\hbar = c = 1$. The starting point for describing double beta decay is the Dirac equation. This relativistic quantum mechanical equation describes free spin $\frac{1}{2}$ particles. The particles involved in double beta decay are quarks, electrons, neutrinos and W bosons. All of these are spin $\frac{1}{2}$ particles, except the W bosons.

2.1 The Dirac equation

The Dirac equation and its conjugate are given by:

$$(i\gamma^\mu \partial_\mu - m\mathbb{1})\psi(x) = 0 \text{ with } (\hbar = c = 1) \tag{2.1}$$

$$\partial_\mu \bar{\psi}(x) i\gamma^\mu + m\mathbb{1}\bar{\psi}(x) = 0 \text{ with } \bar{\psi}(x) = \psi^\dagger(x)\gamma^0 \tag{2.2}$$

In these equations $\psi(x)$ is a four (complex) component Dirac spinor, $\mathbb{1}$ is the four dimensional unit matrix and γ^μ are 4 x 4 matrices for which the following must hold:

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{1} \tag{2.3}$$

$$\gamma^{\mu\dagger} \equiv \gamma^0\gamma^\mu\gamma^0 \tag{2.4}$$

Equation (2.3) is called the Dirac algebra, it has different sets of solutions which are called representations. Both forms of the Dirac equation follow from the Dirac Lagrangian:

$$\mathcal{L}_{Dirac} = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m\mathbb{1})\psi(x) \tag{2.5}$$

From now on, whenever it is more convenient, the four dimensional unit matrix $\mathbb{1}$ and the Minkowski spacetime (x) will be ommited in equations. The Dirac Lagrangian can be split up in a kinetical and massive term:

$$\mathcal{L}_{Dirac} = \mathcal{L}_{kin} + \mathcal{L}_{mass} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \tag{2.6}$$

2.2 Gamma matrices and operators

The gamma matrices, defined by equations (2.3) and (2.4), are not unique. This can be shown. Let U be an unitary matrix, and define the conjugate of γ^μ:

$$\tilde{\gamma}^\mu \equiv U\gamma^\mu U^{-1} \Rightarrow \{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\} = 2g^{\mu\nu}\mathbb{1} \tag{2.7}$$

This fact gives rise to different representations of the Dirac algebra, three of these representations will be discussed in appendix 7.1. In the rest of this section three useful operators will be defined.

2.2.1 The gamma five matrix

The gamma five matrix γ^5 is defined as,

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{2.8}$$

and has three important properties:

$$(\gamma^5)^2 = \mathbb{1} \qquad \gamma^{5\dagger} = \gamma^5 \qquad \{\gamma^\mu, \gamma^5\} = 0 \tag{2.9}$$

2.2.2 The projection operator

The projection operator $P_{\pm}$ is defined as,

$$P^{\pm} \equiv \frac{1}{2}(\mathbb{1} \pm \gamma^5) \tag{2.10}$$

and has three important properties:

$$P^{\pm}P^{\pm} = P^{\pm} \quad P^{\pm}P^{\mp} = 0 \quad P^{+} + P^{-} = \mathbb{1} \tag{2.11}$$

2.2.3 The charge conjugation matrix

The charge conjugation matrix is used to define a charge-conjugate spinor:

$$\psi^c(x) \equiv C\gamma^0\psi^*(x) \tag{2.12}$$

The defining relation for the charge conjugation matrix is:

$$C\gamma^{\mu}C^{-1} = -(\gamma^{u})^T \tag{2.13}$$

If proper normalization for $\psi^c(x)$ is required and $(\psi^c(x))^c = \psi(x)$ two more properties arise:

$$C^{\dagger} = C^{-1} \quad C^T = -C \tag{2.14}$$

The defining relation and the two properties only define C up to an arbitrary phase factor. This means that C can always be multiplied with a factor $e^{i\theta}$ where θ is an arbitrary real number.

2.2.4 The representations

The different representations for the gamma matrices can be found in appendix 7.1.

2.3 Majorana fermions

For a particularly insightful treatment of the definition of a Majorana fermion, chapter 4 of reference [2] can be consulted. Here a shorter version will be given. Define the Majorana fermion through the fact that it is its own antiparticle. This implies that in some way the spinor ψ should be related to ψ^*. The equation $\psi = \psi^*$ is not Lorentz invariant, the solution to this problem is to use the charge-conjugate spinor. This leads to the following definition for a Majorana fermion, which is Lorentz invariant:

$$\psi = e^{i\delta}\psi^c \tag{2.15}$$

Here δ is an arbitrary real number. It is easily seen that this equation, in the Majorana representation, simplifies to:

$$\psi^c_M = C_M\gamma^0_M\psi^*_M = e^{i\theta}(\gamma^0_M)^2\psi^*_M = e^{i\theta}\psi^*_M \tag{2.16}$$

With $\delta = -\theta$ this reduces to:

$$\psi_M = \psi^*_M \tag{2.17}$$

This implies that all 4 components of the Majorana spinor are real, thus in general a Majorana spinor has half as many degrees of freedom as a Dirac spinor.

2.4 The Lorentz group and Weyl spinors

This section and the following section are mainly based on lecture 1 of reference [4], consulting this article is advised. The Lorentz Group, denoted by $SO(3,1)$, is the group of all homogeneous transformations in Minkowski spacetime, $(x^\mu)' = \Lambda^\mu{}_\nu x^\nu$, which leave the length $x^2 = x_\mu x^\mu = x_0^2 - \vec{x}^2$ invariant. The generators of this group satisfy the following algebra:

$$
\begin{aligned}
[R_i, R_j] &= i\epsilon_{ijk} R_k \\
[B_i, B_j] &= -i\epsilon_{ijk} R_k \\
[R_i, B_j] &= i\epsilon_{ijk} B_k
\end{aligned}
\tag{2.18}
$$

Where R_i generate the rotations in three dimensional space and B_i generate the boosts in x, y, z directions. This algebra can be simplified by the linear combinations of R_i and B_i:

$$L_i^{\pm} \equiv \frac{1}{2}(R_i \pm iB_i) \tag{2.19}$$

Now the algebra disentangles to:

$$
\begin{aligned}
\left[L_i^+, L_j^+\right] &= i\epsilon_{ijk} L_k^+ \\
\left[L_i^-, L_j^-\right] &= i\epsilon_{ijk} L_k^- \\
\left[L_i^+, L_j^-\right] &= 0
\end{aligned}
\tag{2.20}
$$

Now it is clear that both generators L_i^+ and L_i^- independently satisfy the $SU(2)$ algebra, thus $SO(3,1)$ is isomorphic to $SU(2) \times SU(2)$. The representations of $SU(2)$ can be labeled by spin, which can take integer and half-integer values. The representation of $SO(3,1)$ is now given by (j_+, j_-), with $j = 0, 1/2, 1, 3/2, ...$

- $(0,0)$ Corresponds to the scalar field / Klein-Gordon equation
- $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ Corresponds to the Weyl spinors / Dirac equation
- $(0, \frac{1}{2}) \oplus (\frac{1}{2}, 0)$ Corresponds to the Dirac spinors / Dirac equation
- $(\frac{1}{2}, \frac{1}{2})$ Corresponds to the vector potential A_μ
- $(1,0) \oplus (0,1)$ Corresponds to the electromagnetic field-strength tensor $F^{\mu\nu}$

A Weyl spinor, χ, corresponding to the $(\frac{1}{2}, 0)$ representation, has generators:

$$
\begin{aligned}
L_i^+ &= \frac{1}{2}\sigma^i \text{ and } L_i^- = 0 \\
R_i &= \frac{1}{2}\sigma^i \text{ and } iB_i = \frac{1}{2}\sigma^i
\end{aligned}
\tag{2.21}
$$

It transforms under rotations, θ, and boosts, $\eta = \operatorname{arctanh}\beta$, as:

$$\chi \to e^{-\frac{i}{2}\sigma\cdot\theta}\chi \text{ and } \chi \to e^{-\frac{1}{2}\sigma\cdot\eta}\chi \tag{2.22}$$

On the other hand, a Weyl spinor corresponding to the $(0, \frac{1}{2})$ representation transforms as:

$$\chi \to e^{-\frac{i}{2}\sigma\cdot\theta}\chi \text{ and } \chi \to e^{\frac{1}{2}\sigma\cdot\eta}\chi \tag{2.23}$$

2.5 Mass terms

The use of Weyl spinors to describe neutrinos is mainly based on the fact that the Dirac equation splits up in two independent equations, if the Weyl spinors describe massless fields. However even if the fields are massive the Weyl spinors are practical to use. The first thing to notice is that a Weyl spinor has 4 degrees of freedom, the same as a Majorana spinor and half as many as a Dirac spinor. In other words, to construct a Majorana spinor only one Weyl spinor is needed and for a Dirac spinor one needs two. First the Dirac and Majorana mass terms will be given in terms of Weyl spinors, with $\epsilon = i\sigma^2$:

$$\begin{aligned}
\mathcal{L}^{D}_{mass} &= m_D \left(\xi^T \epsilon \chi - \chi^\dagger \epsilon \xi^*\right) \\
\mathcal{L}^{M_L}_{mass} &= \frac{1}{2} m_L \left(\chi^T \epsilon \chi - \chi^\dagger \epsilon \chi^*\right) \\
\mathcal{L}^{M_R}_{mass} &= \frac{1}{2} m_R \left(\xi^T \epsilon \xi - \xi^\dagger \epsilon \xi^*\right)
\end{aligned} \tag{2.24}$$

In these equations χ and ξ are the Weyl spinors that both transform under the $(\frac{1}{2}, 0)$ representation of $SO(3,1)$, however $\epsilon\chi^*$ and $\epsilon\xi^*$ transform under the $(0, \frac{1}{2})$ representation. The different mass terms are: $\mathcal{L}^{D}_{mass}$ a Dirac mass, $\mathcal{L}^{M_L}_{mass}$ a Majorana mass for left-handed fermions, $\mathcal{L}^{M_R}_{mass}$ a Majorana mass for right-handed fermions. It is possible to rewrite these mass terms in terms of different spinors, in equations (2.25) - (2.27) these spinors are constructed. These constructions all take place in the Weyl representation, in appendix 7.2.1 some additional properties of that representation are derived. With the help of Weyl spinors, one can construct Dirac and Majorana spinors:

$$\psi_D = \begin{pmatrix} \chi \\ \epsilon\xi^* \end{pmatrix}, \ \psi_{M_L} = \begin{pmatrix} \chi \\ \epsilon\chi^* \end{pmatrix} \text{ and } \psi_{M_R} = \begin{pmatrix} \epsilon\xi^* \\ \xi \end{pmatrix} \tag{2.25}$$

Construct the charge-conjugate spinors in the Weyl representation:

$$\psi^c_D = \begin{pmatrix} \xi \\ \epsilon\chi^* \end{pmatrix}, \ \psi^c_{M_L} = \begin{pmatrix} \chi \\ \epsilon\chi^* \end{pmatrix} \text{ and } \psi^c_{M_R} = \begin{pmatrix} -\epsilon\xi^* \\ -\xi \end{pmatrix} \tag{2.26}$$

It is insightful to notice here that ψ_D indeed has eight degrees of freedom, and both ψ_{M_L} and ψ_{M_R} have four degrees of freedom. Also notice that $\psi_{M_L} = \psi^c_{M_L}$ and $\psi_{M_R} = -\psi^c_{M_R}$ so that the Majorana condition is satisfied for both Majorana spinors. With the help of the projection operator in the Weyl representation, which splits up the Dirac spinor in its chiral components (consult section 4.1.2 for a treatment of helicity and chirality):

$$\begin{aligned}
&\psi_D = P^-_W \psi_D + P^+_W \psi_D = \psi_L + \psi_R \\
&\psi_L = \begin{pmatrix} \chi \\ 0 \end{pmatrix} \qquad \psi_R = \begin{pmatrix} 0 \\ \epsilon\xi^* \end{pmatrix} \\
&(\psi^c)_L = (\psi_R)^c = \begin{pmatrix} \xi \\ 0 \end{pmatrix} \\
&(\psi^c)_R = (\psi_L)^c = \begin{pmatrix} 0 \\ \epsilon\chi^* \end{pmatrix}
\end{aligned} \tag{2.27}$$

It is possible to write the mass terms defined in equation (2.24) in different forms, with the spinors constructed in equations (2.25) - (2.27):

$$
\begin{aligned}
\mathcal{L}_{mass}^{D^1} &= -m_D \bar{\psi}_D \psi_D \\
\mathcal{L}_{mass}^{D^2} &= -m_D \left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right) \\
\mathcal{L}_{mass}^{D^3} &= -m_D \left((\psi^c)_L^T C \psi_L + (\psi^c)_R^T C^T \psi_R \right) \\
\mathcal{L}_{mass}^{D^4} &= -m_D \left(\overline{(\psi^c)}_L m_D (\psi^c)_R + \overline{(\psi^c)}_R m_D (\psi^c)_L \right) \\
\mathcal{L}_{mass}^{M_L^1} &= -\frac{1}{2} m_L \bar{\psi}_{M_L} \psi_{M_L} \\
\mathcal{L}_{mass}^{M_L^2} &= -\frac{1}{2} m_L \left(\psi_L^T C \psi_L + \psi_L^\dagger C^\dagger \psi_L^* \right) \\
\mathcal{L}_{mass}^{M_L^3} &= -\frac{1}{2} m_L \left(\overline{(\psi^c)}_R \psi_L + \bar{\psi}_L (\psi^c)_R \right) \\
\mathcal{L}_{mass}^{M_R^1} &= -\frac{1}{2} m_R \bar{\psi}_{M_R} \psi_{M_R} \\
\mathcal{L}_{mass}^{M_R^2} &= -\frac{1}{2} m_R \left(\psi_R^T C \psi_R + \psi_R^\dagger C^\dagger \psi_R^* \right) \\
\mathcal{L}_{mass}^{M_R^3} &= -\frac{1}{2} m_R \left(\overline{(\psi^c)}_L \psi_R + \bar{\psi}_R (\psi^c)_L \right)
\end{aligned} \tag{2.28}
$$

Here $\bar{\psi}_L$, ψ_L^T, $\psi_L^\dagger$, $(\psi^c)_L^T$ and $\overline{(\psi^c)}_L$ are shorter notations for $\overline{(\psi_L)}$, $(\psi_L)^T$, $(\psi_L)^\dagger$, $((\psi^c)_L)^T$ and $\overline{((\psi^c)_L)}$ respectively. The equivalence of the four Dirac masses, the three left-handed Majorana masses and the three right-handed Majorana masses are derived in appendix 7.2.2. With the newly constructed mass terms, the most general mass term for a spin $\frac{1}{2}$ particle can now be written down. The derivation for this equation can also be found in appendix 7.2.2:

$$
\mathcal{L}_{mass} = \mathcal{L}_{mass}^{D} + \mathcal{L}_{mass}^{M_L} + \mathcal{L}_{mass}^{M_R} \tag{2.29}
$$

$$
\begin{aligned}
\mathcal{L}_{mass} = -\frac{1}{2} \Bigg[&\left(\bar{\psi}_L, \overline{(\psi^c)}_L \right) \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} (\psi^c)_R \\ \psi_R \end{pmatrix} \\
&+ \left(\overline{(\psi^c)}_R, \bar{\psi}_R \right) \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \psi_L \\ (\psi^c)_L \end{pmatrix} \Bigg]
\end{aligned} \tag{2.30}
$$

This is an important result, which will be used in later sections.

3 Properties of the weak interaction

In this section only a few properties of the weak interaction will be addressed. For a general treatment of the Standard Model or the weak interaction the author advises reference [5].

3.1 The electroweak interaction

The electroweak interaction is represented by the gauge group:

$$SU_L(2) \times U_Y(1) \tag{3.1}$$

The subscript L denotes the fact that only left-handed fermions can have weak interactions. The subscript Y indicates the quantum number of weak hypercharge. The generators of this group are:

$$\begin{aligned} &SU_L(2) : W^1_\mu, W^2_\mu, W^3_\mu \\ &U_Y(1) : B_\mu \end{aligned} \tag{3.2}$$

These spin one bosons relate to the physical massive bosons $W^\pm$, Z^0 and the massless photon. The relations between them are given by:

$$\begin{aligned} W^+_\mu &= \frac{1}{\sqrt{2}}\left(W^1_\mu - iW^2_\mu\right) \\ W^-_\mu &= \frac{1}{\sqrt{2}}\left(W^1_\mu + iW^2_\mu\right) \\ Z_\mu &= \cos(\theta_W)W^3_\mu - \sin(\theta_W)B_\mu \\ A_\mu &= \sin(\theta_W)W^3_\mu + \cos(\theta_W)B_\mu \end{aligned} \tag{3.3}$$

The charged-current, neutral-current and electromagnetic Lagrangians are given by:

$$\begin{aligned} \mathcal{L}_{cc} &= ie_W \left[W^+_\mu \bar{\nu}_m \gamma^\mu (1-\gamma^5) e_m + W^+_\mu U_{mn} \bar{u}_m \gamma^\mu (1-\gamma^5) d_n \right. \\ &\qquad \left. + W^-_\mu \bar{e}_m \gamma^\mu (1-\gamma^5)\nu_m + W^-_\mu U^\dagger_{mn} \bar{d}_m \gamma^\mu (1-\gamma^5) u_n\right] \\ \mathcal{L}_{nc} &= \frac{ie}{\sin(\theta_W)\cos(\theta_W)} \sum_f Z_\mu \bar{f}\gamma^\mu \left[(1-\gamma^5)T_3 - Q\sin^2(\theta_W)\right] f \\ \mathcal{L}_{em} &= \sum_f ieA_\mu \bar{f}\gamma^\mu Q f \end{aligned} \tag{3.4}$$

Where $\nu_m = \nu_e, \nu_\mu, \nu_\tau$, $e_m = e, \mu, \tau$, $u_m = u, c, t$, $d_m = d, s, b$ and f denotes any fermion, every spinor in equation (3.4) is a Dirac spinor. In these equations Q and T_3, a $SU(2)_L$ charge component, can be seen as weight factors. For these quantities $Q = T_3 + Y$ must hold. If $Q = 0$, as is the case for neutrinos, then only the charged-current and a part of the neutral-current interactions are existent. From the remaining parts of the Lagrangians one sees that only the left-handed parts of the neutrinos can have weak interaction.

3.2 Conserved quantum numbers

In the standard model there are five conserved quantum numbers corresponding to the invariance of the Lagrangian under five unitary transformations, $U(1)$. The group of these five transformations, denoted by U_{CQN}, is given by:

$$U_{CQN} = U_Q(1) \times U_e(1) \times U_\mu(1) \times U_\tau(1) \times U_B(1) \tag{3.5}$$

In this equation $U_Q(1)$ corresponds to the conservation of electric charge, which is not accidental in the Standard Model. However the other four invariances are accidental, they correspond to the conservation of:

- $U_e(1)$ - electron number : $L_e(e^-) = L_e(\nu_e) = 1$ and $L_e(e^+) = L_e(\bar{\nu}_e) = -1$
- $U_\mu(1)$ - muon number : $L_\mu(\mu^-) = L_\mu(\nu_\mu) = 1$ and $L_\mu(\mu^+) = L_\mu(\bar{\nu}_\mu) = -1$
- $U_\tau(1)$ - tauon number : $L_\tau(\tau^-) = L_\tau(\nu_\tau) = 1$ and $L_\tau(\tau^+) = L_\tau(\bar{\nu}_\tau) = -1$
- $L = L_e + L_\mu + L_\tau$ lepton number: $L(\ell) = L(\nu_\ell) = 1$ and $L(\bar{\ell}) = L(\bar{\nu}_\ell) = -1$ where ℓ denotes a charged lepton.
- $U_B(1)$ - baryon number : $B(q) = \frac{1}{3}$ and $B(\bar{q}) = -\frac{1}{3}$ where q denotes a quark.

Consider for example the $\beta\beta_{2\nu}$ decay of equation (1.2) and Figure $1^{a)}$, let i and f denote the initial and final state respectively:

$$\begin{aligned}
&N_{A,Z} \rightarrow N_{A,Z+2} + 2e^- + 2\bar{\nu}_e \\
&\Delta B = B(f) - B(i) = A - A = 0 \\
&\Delta L_e = L_e(f) - L_e(i) = (2-2) - 0 = 0 \\
&\Delta L = L(f) - L(i) = (2-2) - 0 = 0
\end{aligned} \tag{3.6}$$

As charge is obviously conserved, $\beta\beta_{2\nu}$ decay is allowed by the Standard Model.

3.3 Properties of neutrinos

The neutrino has some general properties and properties which exclusively arise in the Standard Model. Neutrinoless double beta decay can only test some of the Standard Model properties. Below properties of the neutrino are summarized. General Properties:

1. The neutrino is an elementary spin $\frac{1}{2}$ particle with no internal structure.
2. Neutrinos do not carry electric charge.

Standard Model properties:

1. The neutrino has zero mass.
2. The neutrino has no strong and electromagnetic interactions.
3. The neutrino interacts only through the weak interaction.
4. There are only left-handed neutrinos and right-handed antineutrinos.
5. Neutrinos carry lepton number +1 and antineutrinos lepton number -1.

6. Neutrinos do not carry baryon number.

Point one, four and five are related to each other. Because of the two missing helicity states in point four, a Dirac mass term cannot be constructed. The only remaining way to make the neutrinos massive is through the construction of a Majorana mass term. However this is forbidden by lepton number conservation of point five. The fact that Majorana masses violate lepton number conservation will be treated in section 4.2. Thus in the Standard Model neutrinos can not be massive.

4 Majorana neutrinos and $\beta\beta_{0\nu}$ decay

4.1 Requirements for $\beta\beta_{0\nu}$ decay

Neutrinoless double beta decay is not possible in the Standard Model, because of various arguments. In a new theory, which can accomadate $\beta\beta_{0\nu}$ decay, there has to be some physics beyond the Standard Model in which these arguments vanish. The solution which is treated in this text, is the most simple solution. It only modifies the particle content of the Standard Model, and this is only done in the neutrino section. This solution will be called the Majorana solution, referring to the nature of the neutrinos. The various arguments which make $\beta\beta_{0\nu}$ decay impossible and why the Majorana solutions solves these issues, are treated in the next sections.

4.1.1 Lepton number conservation

The major problem of neutrinoless double beta decay is the non-conservation of lepton number. The violation of lepton number by $\beta\beta_{0\nu}$ decay will be shown now, recall equation (1.2) and Figure $1^{b)}$:

$$\begin{aligned} &N_{A,Z} \to N_{A,Z+2} + 2e^- \\ &\Delta B = B(f) - B(i) = A - A = 0 \\ &\Delta L_e = L_e(f) - L_e(i) = 2 - 0 = 2 \\ &\Delta L = L(f) - L(i) = 2 - 0 = 2 \end{aligned} \tag{4.1}$$

Both electron and lepton number are violated by two units. In section 4.2 it will be shown that Majorana mass terms could violate lepton number conservation and therefore can resolve this problem.

4.1.2 Helicity - Chirality

Helicity is the projection of the spin $\vec{S}$ of a particle onto its momentum $\vec{p}$:

$$h = \frac{\vec{S} \cdot \vec{p}}{|\vec{p}|} \tag{4.2}$$

For a spin $\frac{1}{2}$ particle the helicity can be either $+\frac{1}{2}$ or $-\frac{1}{2}$. Particles with $h = +\frac{1}{2}$ are called right-handed and those with $h = -\frac{1}{2}$ are called left-handed. For massless particles helicity is an invariant quantity, however if a particle is massive helicity is not invariant anymore. This occurs if one observer, moving slower than the particle, observes $h = +\frac{1}{2}$. Then a second observer, moving faster than the particle and therefore observes a negative momentum and unchanged direction of the spin, observes $h = -\frac{1}{2}$. For particles with mass, chirality is a more useful quantity. It is a linear combination of helicity states. Consider for example beta decay where only left-handed electrons interact. The electron that in reality is emitted is a combination of both the left-handed and the right-handed state. This can be put in an easy physical model, where the amplitudes depend on the mass and the energy of the electron:

$$\left|e^-\right\rangle = \sqrt{1 - f(E,m)^2}\left|e^-_L\right\rangle + f(E,m)\left|e^-_R\right\rangle \tag{4.3}$$

For ultrarelativistic energies ($E \gg m$): $f(E,m) = (m/E)^2$
For low energies ($E \ll m$): $f(E,m) = \sqrt{\frac{1}{2}}$
Thus the right-handed electron state is greatly suppressed in the ultrarelativistic regime, and non-existent if $m = 0$.
The second problem of neutrinoless double beta decay is helicity conservation. Helicity is conserved in the Standard Model, however this is not the case in $\beta\beta_{0\nu}$decay. This can be illustrated, with the use of Figure 1[b)], in two scenarios:

1. Assume that the neutrino acts as a virtual particle which is created at the upper vertex and absorbed at the lower vertex. Then initially the emitted particle is a right-handed antineutrino, but it has to be absorbed as a left-handed neutrino. This means that helicity is not conserved. It even makes further implications; there has to be some transformation between those neutrinos, which changes a neutrino to an antineutrino or reversed.

2. Assume that two neutrinos are emitted, one at every vertex, which annihilate. Both the emitted particles are right-handed antineutrinos, then annihilation would immediately imply the non-conservation of helicity. A further implication of annihilation would be that either two antineutrinos can annihilate or that the neutrino is its own antiparticle.

The solution to both scenarios is to introduce massive Majorana neutrinos. The fact that the neutrinos are massive resolves the problems with helicity. And the Majorana nature of the neutrinos, i.e. the neutrino is its own antiparticle, fixes the other problems of 1. and 2.

4.2 Conserved quantities

Recall equation (2.24). Let χ and ξ transform under a unitary transformation as:

$$\chi \to U\chi \ , \ \xi \to U^*\xi \text{ and } U^\dagger U = 1 \tag{4.4}$$

Where both χ and ξ still correspond to the $(\frac{1}{2}, 0)$ representation of the Lorentz group. Under these transformations the Dirac mass term remains invariant, opposite to the Majorana mass term which does not. This is shown below:

$$\begin{aligned}
&m_D\left(\xi^T\epsilon\chi - \chi^\dagger\epsilon\xi^*\right) \rightarrow \\
&m_D\left((U^*\xi)^T\epsilon(U\chi) - (U\chi)^\dagger\epsilon(U^*\xi)^*\right) = \\
&m_D\left(\xi^T U^\dagger\epsilon U\chi - \chi^\dagger U^\dagger\epsilon U\xi^*\right) = \\
&m_D\left(\xi^T\epsilon\chi - \chi^\dagger\epsilon\xi^*\right) \\
&\frac{1}{2}m_L\left(\chi^T\epsilon\chi - \chi^\dagger\epsilon\chi^*\right) \rightarrow \\
&\frac{1}{2}m_L\left((U\chi)^T\epsilon(U\chi) - (U\chi)^\dagger\epsilon(U\chi)^*\right) = \\
&\frac{1}{2}m_L\left(\chi^T U^T\epsilon U\chi - \chi^\dagger U^\dagger\epsilon U^*\chi^*\right) \\
&\frac{1}{2}m_R\left(\xi^T\epsilon\xi - \xi^\dagger\epsilon\xi^*\right) \rightarrow \\
&\frac{1}{2}m_R\left((U^*\xi)^T\epsilon(U^*\xi) - (U^*\xi)^\dagger\epsilon(U^*\xi)^*\right) = \\
&\frac{1}{2}m_R\left(\xi^T U^\dagger\epsilon U^*\xi - \xi^\dagger U^T\epsilon U\xi^*\right)
\end{aligned} \tag{4.5}$$

The Majorana mass term is only invariant if $U^T U = 1$ or equivalently $U^\dagger U^* = 1$, which implies that the transformation has to be real. The physical implications of this non-invariance are that a Majorana mass is forbidden if a fermion has non-zero electric charge. A Majorana mass does not only violate conservation of electric charge, but also conservation of the other conserved quantum numbers. The quantities that are possible violated are L_e, L_μ, L_τ, L and B. To summarize one can say that a neutrino can have both Dirac and Majorana masses, because it does not carry electric charge. However if it has a Majorana mass the conservation of the different lepton numbers will be violated.

4.3 Models of neutrino mass

The starting point for neutrino mass is the general mass term of equation (2.30):

$$\begin{aligned}
\mathcal{L}_{mass} = -\frac{1}{2}\Bigg[&\left(\bar{\nu}_L, \overline{(N^c)}_L\right)\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}\begin{pmatrix} (\nu^c)_R \\ N_R \end{pmatrix} \\
&+\left(\overline{(\nu^c)}_R, \overline{N}_R\right)\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}\begin{pmatrix} \nu_L \\ (N^c)_L \end{pmatrix}\Bigg]
\end{aligned} \tag{4.6}$$

Here ν denotes the left-handed neutrino which has Standard Model interactions and N denotes the right-handed sterile neutrino which does not interact with the Standard Model interaction. Both ν and N carry lepton number, so that both mass terms violate lepton number. This mass term describes only one generation of neutrinos, in this case the electron neutrino. The mass model can easily be expanded to three generations, however there is no need for in this thesis. From the requirements for $\beta\beta_{0\nu}$ decay some constraints on the mass matrix can be obtained. There has to be a Majorana mass term to some extent. Thus the only constraint is:

$$m_L > 0 \wedge m_R > 0 \tag{4.7}$$

The eigenvalues of the mass matrix are given by:

$$
\begin{aligned}
& det \begin{pmatrix} m_L - \lambda & m_D \\ m_D & m_R - \lambda \end{pmatrix} = det \begin{pmatrix} m_R - \lambda & m_D \\ m_D & m_L - \lambda \end{pmatrix} \\
& = (m_L - \lambda)(m_R - \lambda) - m_D^2 = 0 \Rightarrow \\
& \lambda = \frac{1}{2}\left(m_L + m_R \pm \sqrt{m_L^2 + m_R^2 + 4m_D^2 - 2m_L m_R}\right)
\end{aligned} \tag{4.8}
$$

4.3.1 Left-handed Majorana mass

In this model $m_L > 0$ and $m_R = m_D = 0$. This seems to be the most simple solution, however it has many disadvantages and is even totally excluded by reference [6]. First consider the mass of the electron neutrino. From various experiments a good estimate is obtained, $m_{\nu_e} < 2\,eV$. This mass should be equal to the only non-vanishing eigenvalue in this model, that is $m_L = m_{\nu_e}$. In this model there is no explanation why m_L is so much smaller than for example the electron mass, $m_e \approx 511\,keV$. The second problem is the fact that although it has a small mass it has never been detected. In physics the detection of particles becomes more difficult as its mass becomes higher or the particle interacts very weakly. The neutrino indeed interacts very weakly, however it has been detected. So if the neutrino is completely of a Majorana nature, one should expect to find more lepton number violating processes. For example the decay of the moun into an electron and a photon, $\mu^- \to e^- + \gamma$. Taking into account reference [6] and these arguments, this model can not explain $\beta\beta_{0\nu}$ decay.

4.3.2 The see-saw mechanism

From the problems in the last section and reference [6] a new model is constructed. In this model $m_L = 0$, $m_R \neq 0$, $m_D \neq 0$ and $m_R \gg m_D$. It is insightful to diagonalize the mass matrix, this is done according to chapter 7.2.4 of reference [2]:

$$
M = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \tag{4.9}
$$

This matrix can be diagonalized with a unitary matrix U:

$$
U = \begin{pmatrix} i\cos\theta & -i\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \text{ with } \tan 2\theta = \frac{2m_D}{m_R} \tag{4.10}
$$

The diagonalized matrix $\mathcal{M}$, with two positive eigenvalues, is obtained by:

$$
\begin{aligned}
& \mathcal{M} = UMU^T = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \text{ where} \\
& m_1 = \frac{1}{2}\left(\sqrt{m_R^2 + 4m_D^2} - m_R\right) \text{ and } m_2 = \frac{1}{2}\left(\sqrt{m_R^2 + 4m_D^2} + m_R\right)
\end{aligned} \tag{4.11}
$$

It is possible to obtain the eigenstates corresponding to these eigenvalues, this is not done here. The two eigenstates are Majorana particles with masses m_1 and m_2. With the condition $m_R \gg m_D$ these masses simplify to:

$$
m_1 = \frac{m_D^2}{m_R} \text{ and } m_2 = m_R \tag{4.12}
$$

Thus by introducing a heavy right-handed neutrino mass one obtains two Majorana particles, a light one and a heavy one. This is called the see-saw mechanism. The particle with m_1 can be interpreted as the electron neutrino, and the particle with m_2 is the sterile neutrino. The mass of these sterile neutrinos can be estimated through relation (4.12). Let $m_1 = m_{\nu_e} < 1\,eV$ and $m_D = m_e \approx 1\,MeV$, then $m_R > 1\,TeV$. Thus the see-saw mechanism can explain why the masses of neutinos and other fermions in the same generation differ so much. And it explains why the Majorana nature of the neutrino has never been detected, because the sterile neutrino responsible for this, has no interactions and is very massive. In contrast to the left-handed Majorana mass model the see-saw mechanism can accommodate neutrinoless double beta decay.

5 The experiment

The experimental setup for neutrinoless double beta decay is straightforward. One starts with a radioactive source, which is known to double beta decay. The major difficulty is how to distinguish between an event where no neutrinos were emitted and an event where two neutrinos were emitted. It is impossible to detect all neutrinos and then compare to the number of electrons emitted, because neutrinos are very hard to detect. The best method to determine which event has taken place, is to look at the energy spectrum of the two electrons. The kinematics of both forms of double beta decay are discussed now.
From chapter 14.2 of reference [2] expressions of the decay rates are obtained. In this derivation the approximation, that the nucleus stays at rest throughout the whole decay, is made. M_I and M_F are the masses of the initial and final state nucleus. $p_i = (E_i, \vec{p}_i)$ are the four momenta of the two electrons and $k_i = \left(\omega_i, \vec{k}_i\right)$ of the neutrinos. Furthermore the following variables are defined:

$$\begin{aligned} T &\equiv Q/m_e = \frac{M_I - M_F - 2m_e}{m_e} \\ y_1 &\equiv \frac{E_1 - m_e}{m_e} \\ y_2 &\equiv \frac{E_2 - m_e}{m_e} \\ y &\equiv y_2 + y_1 = \frac{E_1 + E_2 - 2m_e}{m_e} \end{aligned} \tag{5.1}$$

The decay rates are then given by:

$$\Gamma_{0\nu} \propto \int_0^T dy \int_0^y dy_2 \left|\Re_{0\nu}\right|^2 p_1 p_2 E_1 E_2 \delta(T - y) \tag{5.2}$$

$$\Gamma_{2\nu} \propto \int_0^T dy \int_0^y dy_2 \left|\Re_{2\nu}\right|^2 p_1 p_2 E_1 E_2 (T - y)^5 \tag{5.3}$$

Here The decay rate Γ is a measure for the number of decays per unit time and it is inversely proportional to the mean life time τ. From these equations the major differences between both decays can be obtained, the energy spectrum. In $\beta\beta_{0\nu}$ decay two electrons with energy $E_1 + E_2 = M_I - M_F$ are emitted. In $\beta\beta_{2\nu}$ decay the two electrons can have a range of energies: $0 < E_1 + E_2 < M_I - M_F$. For the double beta decay of ^{76}Ge: $\tau_{2\nu} = 1.55 \cdot 10^{21}\, y$ and as good value $\tau_{0\nu} > 1.5 \cdot 10^{25}\, y$. This gives the following expression for the decay rates:

$$\frac{\Gamma_{2\nu}}{\Gamma_{0\nu}} > 10^4 \tag{5.4}$$

With the help of these constraints a qualitative sketch of the energy spectrum of the two electrons is given in Figure 2, which can be found on the next page. In this sketch Q is the available kinetic energy for the electrons. The focus of the experiment is to measure the little peak at the end of the spectrum, which is caused by $\beta\beta_{0\nu}$ decay. To do this there is need for a significant number of counts in the energy spectrum around the Q-value. Let us examine the experiment of reference [3]. There is a ^{76}Ge source with $N \approx 70\,mol \approx 4 \cdot 10^{25}$ isotopes. The activity $\mathcal{A}$ at $t = 0$ is given by:

$$\mathcal{A} = \frac{N}{\tau_{0\nu}} \approx 2.8\,y^{-1} \tag{5.5}$$

The experiment lasted for ten years, so the expected number of neutrinoless double beta decay events is approximately 25. This is not a significant number if one takes background radiation of ten years into account.

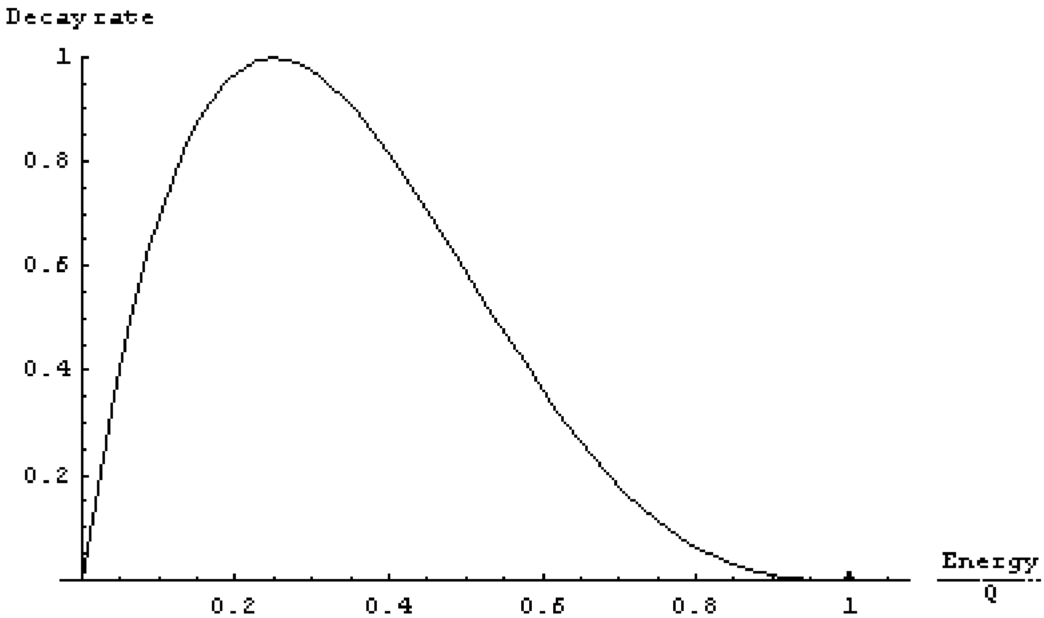

Figure 2: Relative energy spectrum of both double beta decays

6 Conclusions

The author would like to conclude with summarizing the facts that:

- Neutrinoless double beta decay is not allowed in the Standard Model.
- Neutrinoless double beta decay would only be possible if neutrinos are Majorana particles to some extent.
- Neutrinoless double beta decay would only be possible if neutrinos become massive through a Majorana mass term, although Dirac masses are not excluded.
- The see-saw mechanism is the best description for the masses of the neutrinos in this model.
- If Neutrinoless double beta decay would be observed, it would prove that lepton number conservation is violated, that neutrinos are massive and that the neutrino is a Majorana particle to some extent.

The author has no confidence in the conclusions of reference [3], because of the low number of expected events. Follow-up research for this topic could be how the sterile neutrinos become massive in the Higgs model, exploring other models which modify the Standard Model to accomodate neutrino mass or to thorougly investigate the experiment of reference [3].

Aknowledgements

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7 Appendices

7.1 Appendix - Representations for the gamma matrices

For a simple description of the representations, the Pauli matrices will be introduced here:

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

In this section $\mathbb{1}$ will represent the two dimensional unit matrix.

7.1.1 The Dirac representation

The gamma matrices in the Dirac representation are given by:

$$\gamma_D^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \quad \gamma_D^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix} \quad \gamma_D^5 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

The projection operator is given by:

$$P_D^{\pm} = \frac{1}{2}\begin{pmatrix} \mathbb{1} & \pm\mathbb{1} \\ \pm\mathbb{1} & \mathbb{1} \end{pmatrix} = \begin{pmatrix} 1/2 & 0 & \pm 1/2 & 0 \\ 0 & 1/2 & 0 & \pm 1/2 \\ \pm 1/2 & 0 & 1/2 & 0 \\ 0 & \pm 1/2 & 0 & 1/2 \end{pmatrix}$$

The charge conjugation matrix is given by:

$$C_D = e^{i\theta}\gamma_D^0\gamma_D^2 = e^{i\theta}\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}$$

7.1.2 The Weyl representation

The gamma matrices in the Weyl representation are given by:

$$\gamma_W^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \quad \gamma_W^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix} \quad \gamma_W^5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}$$

The projection operator is given by:

$$P_W^+ = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{1} \end{pmatrix} \quad P_W^- = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & 0 \end{pmatrix}$$

The charge conjugation matrix is given by:

$$C_W = e^{i\theta}\gamma_W^0\gamma_W^2 = e^{i\theta}\begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}$$

7.1.3 The Majorana representation

The gamma matrices in the Majorana representation are given by:

$$\gamma_M^0 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix} \quad \gamma_M^1 = \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix} \quad \gamma_M^2 = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix}$$
$$\gamma_M^3 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{pmatrix} \quad \gamma_M^5 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}$$

The projection operator is given by:

$$P_M^\pm = \frac{1}{2}\begin{pmatrix} \mathbb{1} \pm \sigma^2 & 0 \\ 0 & \mathbb{1} \mp \sigma^2 \end{pmatrix} = \begin{pmatrix} 1/2 & \mp i/2 & 0 & 0 \\ \pm i/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & \pm i/2 \\ 0 & 0 & \mp i/2 & 1/2 \end{pmatrix}$$

The charge conjugation matrix is given by:

$$C_M = e^{i\theta}\gamma_M^0 = e^{i\theta}\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}$$

7.2 Appendix - Derivations for section 2

7.2.1 Spinors, projection and charge conjugation

All further derivations will take place in the Weyl representation, some extra properties of ϵ and C_W, with $\theta = \frac{3\pi}{2}$, are given here:

$$\epsilon = i\sigma^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \epsilon^2 = -\mathbb{1} \quad \epsilon^\dagger = \epsilon^T = -\epsilon$$
$$C_W = e^{\frac{3\pi i}{2}}\gamma_W^0\gamma_W^2 = \begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix} \quad C_W^\dagger = C_W^T = -C_W = C_W^{-1}$$

Derivation of the charge conjugates.

$$\psi_D^c = C_W\gamma^0\psi_D^* = \begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix}\begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}\begin{pmatrix} \chi^* \\ \epsilon\xi \end{pmatrix} = \begin{pmatrix} \xi \\ \epsilon\chi^* \end{pmatrix}$$
$$\psi_{M_L}^c = C_W\gamma^0\psi_{M_L}^* = \begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix}\begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}\begin{pmatrix} \chi^* \\ \epsilon\chi \end{pmatrix} = \begin{pmatrix} \chi \\ \epsilon\chi^* \end{pmatrix}$$
$$\psi_{M_R}^c = C_W\gamma^0\psi_{M_R}^* = \begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix}\begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}\begin{pmatrix} \epsilon\xi \\ \xi^* \end{pmatrix} = \begin{pmatrix} -\epsilon\xi^* \\ -\xi \end{pmatrix}$$

7.2.2 Mass terms

Equivalence of the different Dirac mass terms.

$$\begin{aligned}
-m_D\bar{\psi}_D\psi_D &= -m_D\left(\chi^\dagger, -\xi^T\epsilon\right)\begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}\begin{pmatrix} \chi \\ \epsilon\xi^* \end{pmatrix} \\
&= m_D\left(\xi^T\epsilon\chi - \chi^\dagger\epsilon\xi^*\right) \\
-m_D\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right) &= -m_D\left(\left(\chi^\dagger, 0\right)\begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \epsilon\xi^* \end{pmatrix}\right. \\
&\left. + \left(0, -\xi^T\epsilon\right)\begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}\begin{pmatrix} \chi \\ 0 \end{pmatrix}\right) \\
&= m_D\left(\xi^T\epsilon\chi - \chi^\dagger\epsilon\xi^*\right) \\
-m_D\left(\left(\psi^c\right)_L^T C\psi_L + \left(\psi^c\right)_R^T C^T\psi_R\right) &= -m_D\left(\left(\xi^T, 0\right)\begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix}\begin{pmatrix} \chi \\ 0 \end{pmatrix}\right. \\
&\left. + \left(0, \chi^\dagger\epsilon\right)\begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix}\begin{pmatrix} 0 \\ \epsilon\xi^* \end{pmatrix}\right) \\
&= m_D\left(\xi^T\epsilon\chi - \chi^\dagger\epsilon\xi^*\right) \\
-m_D\left(\overline{\left(\psi^c\right)}_L m_D\left(\psi^c\right)_R + \overline{\left(\psi^c\right)}_R m_D\left(\psi^c\right)_L\right) &= -m_D\left(\left(\xi^\dagger, 0\right)\begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \epsilon\chi^* \end{pmatrix}\right. \\
&\left. + \left(0, -\chi^T\epsilon\right)\begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}\begin{pmatrix} \xi \\ 0 \end{pmatrix}\right) \\
&= m_D\left(\chi^T\epsilon\xi - \xi^\dagger\epsilon\chi^*\right)
\end{aligned}$$

Equivalence of the different left-handed Majorana mass terms.

$$\begin{aligned}
-\frac{1}{2}m_L\bar{\psi}_{M_L}\psi_{M_L} &= -\frac{1}{2}m_L\left(\chi^\dagger, -\chi^T\epsilon\right)\begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}\begin{pmatrix} \chi \\ \epsilon\chi^* \end{pmatrix} \\
&= \frac{1}{2}m_L\left(\chi^T\epsilon\chi - \chi^\dagger\epsilon\chi^*\right) \\
-\frac{1}{2}m_L\left(\psi_L^T C\psi_L + \psi_L^\dagger C^\dagger\psi_L^*\right) &= -\frac{1}{2}m_L\left(\left(\chi^T, 0\right)\begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix}\begin{pmatrix} \chi \\ 0 \end{pmatrix}\right. \\
&\left. + \left(\chi^\dagger, 0\right)\begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix}\begin{pmatrix} \chi^* \\ 0 \end{pmatrix}\right) \\
&= \frac{1}{2}m_L\left(\chi^T\epsilon\chi - \chi^\dagger\epsilon\chi^*\right) \\
-\frac{1}{2}m_L\left(\overline{\left(\psi^c\right)}_R\psi_L + \bar{\psi}_L\left(\psi^c\right)_R\right) &= -\frac{1}{2}m_L\left(\left(0, -\chi^T\epsilon\right)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \chi \\ 0 \end{pmatrix}\right. \\
&\left. + \left(\chi^\dagger, 0\right)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \epsilon\chi^* \end{pmatrix}\right) \\
&= \frac{1}{2}m_L\left(\chi^T\epsilon\chi - \chi^\dagger\epsilon\chi^*\right)
\end{aligned}$$

Equivalence of the different right-handed Majorana mass terms.

$$
\begin{aligned}
-\frac{1}{2}m_R\bar{\psi}_{M_R}\psi_{M_R} &= -\frac{1}{2}m_R\left(-\xi^T\epsilon,\xi^\dagger\right)\begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}\begin{pmatrix} \epsilon\xi^* \\ \xi \end{pmatrix} \\
&= \frac{1}{2}m_R\left(\xi^T\epsilon\xi - \xi^\dagger\epsilon\xi^*\right) \\
-\frac{1}{2}m_R\left(\psi_R^T C\psi_R + \psi_R^\dagger C^\dagger\psi_R^*\right) &= -\frac{1}{2}m_R\left(\left(0,-\xi^\dagger\epsilon\right)\begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix}\begin{pmatrix} 0 \\ \epsilon\xi^* \end{pmatrix}\right. \\
&\quad\left. + \left(0,-\xi^T\epsilon\right)\begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix}\begin{pmatrix} 0 \\ \epsilon\xi \end{pmatrix}\right) \\
&= \frac{1}{2}m_R\left(\xi^T\epsilon\xi - \xi^\dagger\epsilon\xi^*\right) \\
-\frac{1}{2}m_R\left(\overline{(\psi^c)}_L\psi_R + \bar{\psi}_R\left(\psi^c\right)_L\right) &= -\frac{1}{2}m_R\left(\left(\xi^\dagger,0\right)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \epsilon\xi^* \end{pmatrix}\right. \\
&\quad\left. + \left(0,-\xi^T\epsilon\right)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \xi \\ 0 \end{pmatrix}\right) \\
&= \frac{1}{2}m_R\left(\xi^T\epsilon\xi - \xi^\dagger\epsilon\xi^*\right)
\end{aligned}
$$

Derivation of the total mass term.

$$
\begin{aligned}
\mathcal{L}_{mass} &= -\frac{1}{2}\left[\left(\bar{\psi}_L,\overline{(\psi^c)}_L\right)\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}\begin{pmatrix} (\psi^c)_R \\ \psi_R \end{pmatrix}\right. \\
&\quad\left. + \left(\overline{(\psi^c)}_R,\bar{\psi}_R\right)\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}\begin{pmatrix} \psi_L \\ (\psi^c)_L \end{pmatrix}\right] \\
&= -\frac{1}{2}\left[\bar{\psi}_L m_L\left(\psi^c\right)_R + \bar{\psi}_L m_D\psi_R + \overline{(\psi^c)}_L m_D\left(\psi^c\right)_R + \overline{(\psi^c)}_L m_R\psi_R\right. \\
&\quad\left. + \overline{(\psi^c)}_R m_L\psi_L + \overline{(\psi^c)}_R m_D\left(\psi^c\right)_L + \bar{\psi}_R m_D\psi_L + \bar{\psi}_R m_R\left(\psi^c\right)_L\right] \\
&= \mathcal{L}_{mass}^{M_L^3} + \frac{1}{2}\mathcal{L}_{mass}^{D^2} + \frac{1}{2}\mathcal{L}_{mass}^{D^4} + \mathcal{L}_{mass}^{M_R^3} = \mathcal{L}_{mass}^{D} + \mathcal{L}_{mass}^{M_L} + \mathcal{L}_{mass}^{M_R}
\end{aligned}
$$

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