Rijks*universiteit* Groningen

Massive Neutrinos and their Occurrence in a Left-Right Symmetric Model

author
Duurt Johan van der Hoek

supervisor
Prof. Dr. M. de Roo

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CHAPTER 1

Introduction

1.1 General Introduction

One can argue about which achievement in physics is the greatest of all. Some would argue that Newton's laws of classical mechanics or the unification of electricity and magnetism deserves this title. But certainly a good candidate would be the *standard model of elementary particles.* It is a quantum field theory embodying all known particles and their interactions. For many years it has proven to provide us with an extremely accurate tool to describe these interactions. Although the standard model does a very good job, from the beginning people realized it was not perfect, in the sense that it was not a complete description of nature. The most obvious shortcoming is that gravity is left out. Another, less obvious, problem goes by the name *hierarchy problem* [1] and also indicates the existence of physics beyond the standard model. Not too long ago another important thing turned out to be missing. It has everything to do with the neutrino; the particle that is said to permeate our bodies at a staggering rate of many billions per second!

In 1930, long before the formulation of the standard model, Wolfgang Pauli was the first to imagine the existence of this neutral particle and called it the neutron. He was led to this idea in an attempt to address the missing energy and angular momentum in the process of beta decay in which a neutron decays into a proton, an electron and, he reasoned, a neutrino. A few years later Enrico Fermi formulated the famous Fermi theory of the weak interactions and changed the name of the still undetected particle from neutron into neutrino. Still at this stage the concept of a vector boson as a force agent was unknown. In the forties this was introduced by (among others) Richard Feynman in quantum electrodynamics, where the photon plays the role of a force carrier. Physicists realized the Fermi theory had its shortcomings and, inspired by the structure of QED, tried to build a combined model model of electromagnetic and weak interactions, resulting in the famous $SU(2) \times U(1)$ gauge theory. Only massless particles could be described by the model, which was eventually cured in the late sixties by the implementation of the Higgs-mechanism and resulted in the famous GSW-model of electro-weak interactions, named after Sheldon Glashow, Abdus Salam and Steven Weinberg.

Sofar this very short history of the electroweak part of the standard model. As said, the neutrino was first 'made up' in the thirties, but because of its zero electric (and clearly, color-) charge it only feels the weak interactions and it was not until 1956 that it was actually detected by Frederick Reines and Clyde Cowan. Again 40 years later the 1995 nobel prize was awarded for their discovery. The neutrino mentioned above is associated with the electron and called the electron neutrino. After the discovery in 1962 of a second type of neutrino in reactions involving muons (nobel prize 1988) people expected a third corresponding to the tau lepton and, indeed, in 2000 it was first observed at Fermilab in nice accordance with the standard model.

At the time the standard model acquired its final shape, the neutrino was considered massless and described accordingly. Given the gauge group, the fieldcontent and their transformation properties under the gauge group, it turns out that the neutrino cannot possibly be described as a massive particle. Preserving the standard model gauge group, we either need to add new fermion fields or scalar fields for neutrino mass terms to appear.

When the standard model was formulated physicists did everything but lay back, enjoying their masterpiece. Models were invented in which the unanswered question of maximally broken parity in the weak interaction was addressed and attempts were made to further unify strong and weak interactions but also leptons and quarks. Meanwhile, often in a combination with these two issues, models that allowed for massive neutrinos were under investigation because, after all, the only certain thing about neutrino masses was that they had to be smaller than a few electron volts.

In 1998 the physics community saw direct evidence of the existence of neutrino masses, in the shape of observed neutrino oscillations in the Superkamiokande collaboration based in Japan. As early as 1957 this phenomenon had been hypothesized by Bruno Pontecorvo. The only driving force behind these oscillations, so far as people know nowadays, is mass or, more correctly, mass differences. The oscillations reveal information on the mass difference between the particles involved. There are other types of experiments from which we can learn more about the absolute scale of the mass.

In short: the neutrino mass *is* there, the standard model *is* extremely accurate but no doubt incomplete. In this report a specific model will be explored that allows for the presence of neutrino masses and is able to account for their smallness. Also, it deals with the odd asymmetry between left and right in the weak interactions.

1.2 This Report

The outline of this report is as follows

CHAPTER 2. Before one should start thinking about extensions of the standard model, it is good to have some understanding of the standard model itself. A requirement of the new model is that at low energies it reproduces the standard model, up to some corrections suppressed by the large masses of some inevitable new particles. In this chapter the electro-weak sector of the standard model will be reviewed and in of course the role neutrino is discussed.

CHAPTER 3. The standard fermion mass terms like the one for the electron is known as a *Dirac mass.* A right handed field is coupled to a left handed field. There is a second type of mass term called *Majorana mass* term. In this case, left is coupled to left and right to right. An inevitable consequence is that lepton number will be violated. To be able to explain all this, the chapter will start out by introducing the two component Weyl spinors; the building blocks for the Dirac and Majorana spinor.

CHAPTER 4. Some general aspects of massive neutrino physics will be treated. A short introduction on the history of neutrino related experiments is followed by an application of the ideas of chapter 3 to neutrinos. The seesaw mechanism will be explained as well as some aspects of neutrino mixing and the simplest extensions of the standard model are discussed.

CHAPTER 5. Until this chapter all aspects of non-vanishing neutrino masses are discussed outside the context of a concrete model. Here an attractive extension of the standard model is discussed in quite some detail: a left-right symmetric model with one bi-doublet and two triplet scalar fields. This set of scalars is the smallest able to produce the most general neutrino mass terms already at tree level. We will see that a seesaw mechanism is more or less automatically incorporated.

CHAPTER 6. In this chapter a humble attempt is made to get a glimpse at some numerical values of the main parameter in the model: the mass scale of the extra gauge bosons, the right handed neutrinos and some Yukawa coupling strengths. This is checked against some experimental results. The chapter closes with a remark on neutrinoless double beta decay.

CHAPTER 2

Neutrinos in the Standard Model

In this report the neutrino masses are of main interest. Before one looks at possible extensions of the standard model, it is vital to understand the story of the neutrino in the standard model. In chapter 3 two different kinds of possible mass terms are discussed. The construction of the standard model was, of course, motivated by experimental results that led to the exile of the right handed part of the neutrino (more on this later). There was simply no need for it. Consequently, the Dirac type of mass is ruled out. Also, the other type, the Majorana mass, is no possibility in the standard model because the left handed neutrino carries a $U(1)$ charge. These remarks will become clear in this and the next chapter.

In the following the leptonic electro-weak sector of the standard model, is reviewed. The quarks and their weak as well as their strong interactions will be left out of the discussion.

2.1 The Electro-Weak Model and the Absence of a Right Handed Neutrino

The standard model is a gauge theory with symmetry group $SU(2)_L \times U(1)_Y$ resulting in the $W^{\pm}$ and Z^0 gauge fields. Around 1972 it was realized that this could solve the problems at high energy of the Fermi theory, being *the* description of the weak interaction until then. Some authors take this group structure as some axiom and start from there. The reasoning may be 'clean' and beautiful, but not very intuitive: why would one take this as a starting point? How can one justify this choice of the gauge group other than 'it works'?

In the strategy described above one uses Noether's theorem, guided by the *imposed* symmetries, to find the conserved currents and the corresponding charges. It may seem more natural that we do not know the symmetry group, but we may have some knowledge about the currents from experiments. This allows us to work in the other direction (see for example [2, 3]): one knows the current and hence the charges. Then realize that the charges satisfy the same commutation relations as the generators responsible for them. In other words: the commutation relation of the charges uncover the structure of the underlying group.

In '57 it was first proposed by Feynman and Gell-Mann that [4]

$$j^\mu = \overline{\psi}_\nu \gamma^\mu (1-\gamma^5)\psi_e \tag{2.1}$$

was the correct description of the charged current. For the μ and τ generations exactly the same holds.

Before we calculate the charges and their commutation relations, a few remarks about this current are in place. The factor $(1-\gamma^5)$ makes sure only the lefthanded parts of the fields (ψ_L) take part in the interaction:

$$\psi_L := P_-\psi := \tfrac{1}{2}(1-\gamma^5)\psi \tag{2.2}$$

The righthanded part (ψ_R) is defined analogously but with pluses. The operators $P_\pm$ are projection operators in the usual sense: $P_+ + P_- = 1$, $P_\pm^2 = P_\pm$ and $P_\pm P_\mp = 0$. It is clear at first sight that in (2.1) only the lefthanded part of the electron is present. To see that the same holds for the neutrino, write ψ_ν as the sum of its left- and righthanded part and applying the rules of appendix A will lead us to the conclusion that the neutrino can just as well be replaced by its lefthanded part.

At this point it is already possible to see why the righthanded part of the neutrino was not included in the standard model: under the dynamics of the Dirac equation 'left remains left' and so does right. This is only true for mass*less* particles[1] and since the neutrino was regarded massless at the time, the righthanded part would be completely decoupled from everything else. So why bother to take it along in the description? To put it more dramatically, its very existence could be questioned.

[1]Consequently $\partial_\mu(P_\pm\psi) = 0$ for zero mass particles so that currents such as $j^\mu = \bar{\psi}_L\gamma^\mu\psi_L$ will be conserved, i.e. $\partial_\mu j^\mu = 0$.

Let us follow the programme outlined above to obtain the group structure of the theory of the weak interactions. With (2.1) we can write down the following charges:

$$\begin{aligned} T_+ &= \tfrac{1}{2}\int d^3x\, j^0 = \int d^3x\, \psi^\dagger_{\nu,L}\psi_{e,L} \\ T_- &= T_+^\dagger = \tfrac{1}{2}\int d^3x\, j^{0\dagger} = \int d^3x\, \psi^\dagger_{e,L}\psi_{\nu,L} \\ T_3 &= \tfrac{1}{2}[T_+, T_-] = \tfrac{1}{2}\int d^3x\, (\psi^\dagger_{e,L}\psi_{e,L} - \psi^\dagger_{\nu,L}\psi_{\nu,L}) \\ Q &= -\int d^3x\, \psi^\dagger_e\psi_e \end{aligned} \tag{2.3}$$

Where the last one is nothing more than the electric charge. When we make the following redefinitions:

$$\begin{aligned} Q_1 &= \tfrac{1}{2}(T_+ + T_-) \\ Q_2 &= \tfrac{1}{2i}(T_+ + T_-) \\ Q_3 &= T_3 \\ Y &= 2(Q - T_3) \end{aligned} \tag{2.4}$$

and calculate the commutators between these charges one can indeed see the group structure appear.

$$\begin{aligned} [Q_a, Q_b] &= i\epsilon_{abc}Q_c \\ [Q_a, Y] &= 0 \end{aligned} \tag{2.5}$$

The first line of (2.5) shows that we are dealing with $SU(2)$. Besides there is also second group - $U(1)$ - with one generator: the so-called hyper charge Y.

The above shows how one arrives at the particular group choice. We know that the symmetries come with conserved charges, so it is important to be sure they are indeed conserved (ie. $\partial_\mu j^\mu = 0$). To check this, we need to invoke the Dirac equation and it turns out that only for a vanishing electron mass there will be charge conservation (see previous footnote). For that reason all the fermion masses will be set to zero at this stage.

The zero mass of all fermions and the non-existence of the right handed neutrino brings us to the following Lagrangian:

$$\mathcal{L} = \overline{L}i\partial\!\!\!/ L + \overline{e}_R i\partial\!\!\!/ e_R \tag{2.6}$$

From now on we will denote the fermion fields by some suitable letter instead of ψ carrying a zoo of indices. In particular $e_R := \psi_{e,R}$. Also, in (2.6), we have defined the left handed lepton doublet:

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \tag{2.7}$$

This L is a doublet of $SU(2)$ whereas e_r is a singlet. The Y charge of the doublet is -1 and of e_R -2. The important thing is that (2.6) is invariant under the action of $SU(2) \times U(1)$. The transformation rule for the doublet will be $L \to e^{i\alpha^a \tau^a} e^{i\beta Y/2} L$ where the Y in the second exponent can be replaced by the *number* representing the Y-charge of the object of interest.

Now, the next step is to promote the global symmetry to a local one. In doing so, one gauge field for every generator of the group is needed. In this case it gives three corresponding to $SU(2)$, one to $U(1)$. The ordinary derivatives are replaced by covariant derivatives. For some generic symmetry group with generators T_a we have the replacement (see appendix B)

$$\partial_\mu \to D_\mu = \partial_\mu - igT^a A^a \tag{2.8}$$

In the particular case of the lepton doublet L and the right handed electron e_R this implies[2]:

$$\begin{aligned} D_\mu L &= (\partial_\mu - igA^a_\mu \tau^a + \tfrac{1}{2} ig' B_\mu) L \\ D_\mu e_R &= (\partial_\mu + ig' B_\mu) e_R \end{aligned} \tag{2.9}$$

The above replacement of the ordinary derivative by the covariant version gives rise to interaction terms of the fermions with the gauge fields A^a_μ and B_μ. Clever redefinitions of the A and B fields make sure that, for example, there is a vector field that couples to the electron, but does not 'see' the neutrino. In fact, it has the correct couplings known from QED[3] to be regarded as the photon field.

Sofar, the dynamics of the 'force agents'[4] themselves have not been mentioned. It will not be of much importance here, but for completeness, the kinetic term for the gauge fields are (see for example [5])

$$\mathcal{L}_{kin} = -\tfrac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} \quad \text{with} \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^a_{\ bc} A^b_\mu A^c_\nu \tag{2.10}$$

[2] We apply the widely used notation $\tau^a = \sigma^a/2$.

[3] Plus some extra couplings with the 'new' charged gauge fields.

[4] As 't Hooft tends to call gauge fields.

In this equation the numbers $f^a_{\ bc}$ are the structure constants of the group corresponding to the gauge fields A^a_μ. This is the origin of vertices involving two or three gauge fields, that are so characteristic for non abelian gauge theories.

2.2 The Higgs Mechanism

Sofar, all fields are massless. One cannot write down a mass term without violating gauge invariance. It was realized that the introduction of some new scalar fields could save the day. The Lagrangian of the field and its couplings respect the gauge symmetry, but its ground state *does* break gauge invariance leading to mass terms via Yukawa couplings of the scalar fields to two fermion fields. In the process, the gauge fields also acquire a mass. This spontaneous symmetry breaking[5] can be compared with a pencil balancing on its point on a table. The Hamiltonian of the system is cylindrically symmetric. However, in the ground state the pencil lies flat on the table pointing in some specific direction. Here we will see how this idea is put into the standard model.

A 'standard' electron mass term (classified as a Dirac mass in the next chapter) looks like

$$\mathcal{L}_m = -m\bar{\psi}_e\psi_e = -m(\bar{e}_L e_R + \bar{e}_R e_L). \tag{2.11}$$

Clearly, this is not invariant under the group action: under an $SU(2)$ transformation, e_L and ν_L go into linear combinations of each other and equation (2.11) will definitely change. Also, the $U(1)$ transformation causes trouble.

A solution to this problem lies in the introduction of an $SU(2)$ doublet of scalar fields

$$\phi = (\phi^1, \phi^2)^T \tag{2.12}$$

having $Y = 1$ so that the gauge invariant Yukawa coupling between these scalar fields and the fermions can be formed

$$\mathcal{L}_Y = f\bar{L}\phi e_R + h.c. \tag{2.13}$$

If the Lagrangian looks like

[5]Gerard 't Hooft argues that spontananeous symmetry breaking is in fact a misnomer. No symmetry is broken, but the physical particles are not in a representation of the gauge group. The local symmetry is merely hidden by the shift in the scalar field. [7]

$$\mathcal{L}_H = (D_\mu\phi)^\dagger D^\mu\phi + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2$$

$$\text{with} \quad D_\mu\phi = (\partial_\mu - ig\tau^a A^a_\mu - \frac{ig'}{2}B_\mu) \tag{2.14}$$

the last two terms, being the Higgs potential, cause the doublet to have a non-zero vacuum expectation value: the minimum of the potential is at $\phi^\dagger\phi = \mu/\sqrt{\lambda} := v^2 \neq 0$. Choosing[6] $\langle\phi\rangle := \langle 0|\phi|0\rangle = (0, v)^T$ we can parametrize the field as $\phi = U(0, v+\phi')^T$, where U is and $SU(2)$ group element, parametrized as $U = \exp[i\xi^i\tau^i]$ so the ϕ doublet still has four degrees of freedom, as it started out with in (2.12). We can do an $SU(2)$ gauge transformation to get rid of this matrix U and hence work in the so called unitary gauge. The parametrization of the scalar doublet we are left with is just $(0, v + \phi')$. The three degrees of freedom, that seem to have disappeared, are by some people [6] said to be eaten by the gauge bosons which have acquired a mass in this procedure and, therefore, gained a degree of freedom.

2.2.1 Gauge Boson Masses

The exact expressions for the gauge boson masses follow directly when substituting $\langle\phi\rangle$ for ϕ in (2.14). We single out the terms quadratic in the gauge fields:

$$\mathcal{L} = \frac{v^2}{4}\begin{pmatrix} A^1_\mu \\ A^2_\mu \\ A^3_\mu \\ B_\mu \end{pmatrix}^T \begin{pmatrix} g^2 & & & \\ & g^2 & & \\ & & g^2/2 & -gg'/2 \\ & & -gg'/2 & g'^2/2 \end{pmatrix} \begin{pmatrix} A^{1,\mu} \\ A^{2,\mu} \\ A^{3,\mu} \\ B^\mu \end{pmatrix}. \tag{2.15}$$

where empty entries are zero. The eigenvectors tell what are the massive combinations of the gauge fields and the corresponding eigenvalues give the masses through[7] $m^2 = 2 \times$ eigenvalue. For example, (1,0,0,0) is obviously an eigenvector with eigenvalue $(gv)^2/4$. So we conclude that A^1_μ has a mass $gv/\sqrt{2}$. The full list reads[8]:

[6] In fact, any choice $\langle\phi\rangle = (v1, v2)^T$ with $|v_1|^2 + |v_2|^2 = v^2$ will do the job. The mass terms that arise will not be diagonal in the fermion fields and a redefinition will be necessary to fix this. At the end of the run it is equivalent to choosing $\langle\phi\rangle = (0, v)^T$.

[7] This is because the mass terms look like $\frac{1}{2}M^2A^2$. This implies that the mass term for $W^\pm = \frac{1}{\sqrt{2}}(A^1 \mp iA^2)$ is lacking the factor $\frac{1}{2}$ and looks like $M^2W^+W^-$.

[8] In many textbooks people use and alternative definition of the vev v, namely $v_{there} = v_{here}/\sqrt{2}$. See for example Peskin & Schroeder [5] ch. 20.

$$
\begin{array}{lll}
W^{\pm} = \frac{1}{\sqrt{2}}(A^1 \mp iA^2); & & m_W = gv/\sqrt{2} \\
Z = \cos\theta_w A^3 - \sin\theta_w B; & & m_Z = \sqrt{g^2 + g'^2}v/\sqrt{2} \\
A = \sin\theta_w A^3 + \cos\theta_w B; & & m_A = 0
\end{array}
\tag{2.16}
$$

A few things need to be said about this. Firstly, the cosine of the angle θ_w, known as the Weinberg angle, is defined by $\cos\theta_w = g/\sqrt{g^2 + g'^2}$ so that $\sin\theta_w = g'/\sqrt{g^2 + g'^2}$. Secondly, since A^1 and A^2 are 'eigenvectors' with the same eigenvalue, any linear combination of the two is again one with this eigenvalue. The first two lines in the table give convenient linear combinations that ensures we have two massive *charged* gauge bosons. Lastly, we note that the following relation holds: $m_W = m_Z \cos\theta_w$.

We are now in a position to estimate the numerical value of some of the couplings g and g' and v. From measurements it is known that $m_w = gv/\sqrt{2} = 80.4$ GeV and $m_z = 91.1$ GeV which gives $\sin^2\theta_w = 0.22$. This already tells us that $g/g' = \cot\theta_w = 1.7$. To obtain an absolute scale we merely mention that we are forced[9] to make the identification $gg'/\sqrt{g^2 + g'^2} = g\sin\theta = e$, the coupling constant of the electromagnetic interaction. In natural units, via $137 = \alpha^{-1} = 4\pi/e^2$, we find $e = 0.30$, giving in turn $g = 0.64$ and $g' = 0.38$. With the mass formula for the W boson this finally gives $v \approx 1.8 \times 10^2$ GeV. This is often referred to as 'the weak scale'.

2.2.2 Lepton Masses

What we are also interested in is what the Higgs mechanism does to the fermions. For these fields it is fairly easy to read off the masses from the Yukawa terms. Substituting $\phi = (0, v + \phi')^T$ in (2.13) we see a mass term arising for the electron:

$$
\mathcal{L}_m = vf\bar{e}_L e_R + h.c. \tag{2.17}
$$

The Yukawa coupling f must be very small ($\sim 10^{-6}$) in order to give the electron mass $m_e = vf$ its, compared to the weak scale, small value of 0.5 MeV.

Without the presence of ν_R in the standard model it is not possible to construct a mass term as in (2.17) for the neutrino. In the next chapter we will see that a different kind of mass term can be constructed ($-\frac{1}{2}m(\nu_L^T C\nu_L + h.c.)$), involving only ν_L. However, this is forbidden within the standard model since it is not invariant under the $U(1)_Y$ symmetry.

[9] This will become clear in the first stage of symmetry breaking in the model described in chapter 5, where something very similar takes place.

2.3 Lepton Number Conservation

From experiment it is known that lepton number is conserved to a high degree of accuracy (for example [8]). To be more precise, there appears to be lepton number conservation within each generation. The way this is apparent in the SM is through an accidental symmetry in the shape of the global $U(1)$ transformation

$$\begin{aligned} L &\rightarrow e^{i\theta} L \\ e_R &\rightarrow e^{i\theta} e_R, \end{aligned} \tag{2.18}$$

which leaves the Lagrangian invariant. Later we will see that the so-called Majorana mass term (see section 3.3) disturbs the invariance under (2.18). The presence of such terms can result in some new physical processes, described briefly in section 4.1.

2.4 Remark on describing more than one Generation

2.4.1 Leptons

We close this chapter with a final note. In the entire chapter we merely mentioned the muon, the tauon and their corresponding neutrinos. These other two lepton generations can be very easily incorporated in the SM by just duplicating all terms for every generation. In practice this is usually done by giving the lepton doublet a 'generation index' index: L is replaced by L^i so that the kinetic term becomes $\bar{L}^i \not{D} L^i$ where there is a sum over the generations. Note that all generations couple with the same strength to the gauge fields.

A more profound effect of adding two more generations happens through the Yukawa couplings. There is a priory no reason for them to be diagonal. Equation (2.13) is replaced by[10]

$$\mathcal{L}_Y = f^{ij} \bar{L}^i \phi e_R^j + h.c.$$

Where the complex 3×3 matrix f is not constrained by any symmetry. This seems to add the undesirable amount of $3^2 \times 2 = 18$ free parameters to the theory. Fortunately, there is the accidental (i.e. not explicitly imposed) global the $[U(3)]^2$ symmetry

[10]Notation $e_R^i = e_R, \mu_R, \tau_R$.

$$L^i \to U_L^{ij} L^j \quad \text{and} \quad e_R^i \to U_{e_R}^{ij} e_R^j, \tag{2.19}$$

which effectively replaces f by $f' = U_L f U_{e_R}^\dagger$. So it seems that by choosing the transformations properly, f can be brought to a more simple form with less free parameters.

How much simpler can it be made? If we put f in some restricted form by using the freedom expressed in equation (2.19), we clearly no longer have the (full) $[U(3)]^2$ symmetry. Now, note that (2.19) is the three generation equivalent of the transformation related to lepton number conservation (2.18) if $U_{e_R} = U_L$. What we would like to end up with is a symmetry (also in the Yukawa Lagrangian) that guarantees the lepton number conservation per generation. This is accomplished by invariance under a 3-parameter subgroup of (2.19): $L^i \to e^{i\theta_i} L^i$ and $e_R^i \to e^{i\theta_i} e_R^i$; all generations transform independently. This subgroup of $[U(3)]^2$ we cannot use to simplify f, so we only have $2 \times 3^2 - 3 = 15$ parameters we *can* use. In its simplest and yet most general form, f has only three (18-15) free parameters, all of which are needed to set the values of the masses. In other words, f can be put in *real* diagonal[11] form, sacrificing the freedom of (2.19). We conclude that the gauge eigenstates coincide with the mass eigenstates.

2.4.2 Quark Mixing

When we do a similar counting in the quark sector something very different happens. Without going into details, it turns out there are 10 free parameters. This is reflected in the fact that there are 6 quark masses and 4 parameters to be chosen in the famous CKM-matrix. Three of them are mixing angles, one is a CP-violating phase. The CKM-matrix enters the theory through the charged current Lagrangian which looks schematically like

$$\bar{u}_L^i \gamma^\mu U_{CKM}^{ij} d_L^j W_\mu^+$$

It is clear that off-diagonal elements of U_{CKM} cause vertices with quarks from different generations. In section 4.2.2 we will encounter almost an exact equivalent for the lepton sector, which goes by the name of NMS-matrix.

Note that the masslessness of the neutrinos imply that a CKM-like matrix cannot be present in the lepton sector. If the gauge eigenstates of the charged leptons *were* different from the mass eigenstates, they *would* require a redefinition (see for more details in section 4.2.2). If the neutrino fields are

[11] In fact, it must be diagonal to guarantee lepton number conservation for each generation separately.

redefined by exactly the same transformation, which can be done without consequences because of the absence of mass terms, the equivalent of the CKM-matrix in the above expression would simply be the unit matrix.

CHAPTER 3

Dirac and Majorana Mass Terms

In this chapter different types of fermion mass terms will be discussed. In the previous section 2 the leptons gained their masses through terms like:

$$\mathcal{L}_{\text{mass}} = -m\overline{\psi}\psi = -m(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L) \tag{3.1}$$

Besides this so called Dirac mass there is the Majorana mass term. In order to define it, it is useful to introduce the Weyl spinor. We will see different ways to combine these objects to form Lorentz invariant terms involving both one and two different Weyl spinors to form Majorana and Dirac masses.

3.1 Weyl spinors

One can roughly say that the upper (and lower) two components of a four-componenent spinor compose one Weyl spinor. To be more precise we define the Weyl spinor according to its transformation behavior under the Lorentz group, $SO(3,1)$. Its generators can be divided in boosts and rotations, K_i and J_i respectively. Of course there are three of each. The following commutation relations between them hold[1]:

[1] According to Anthony Zee the second line of (3.2) represents "one of the most significant calculations in the history of twentieth century physics. (...) Two Lorentz boosts produce a rotation!" [6]

$$
\begin{aligned}
[J_i, J_j] &= i\epsilon_{ijk} J_k \\
[K_i, K_j] &= -i\epsilon_{ijk} J_k \\
[J_i, K_j] &= i\epsilon_{ijk} K_k
\end{aligned} \tag{3.2}
$$

From this set of generators we can define $A_i = \frac{1}{2}(J_i + iK_i)$ and $B_i = \frac{1}{2}(J_i - iK_i)$. It is important to note that the (anti-)hermicity of the (K_i) J_i ensures that A_i and B_i are hermitian. Using the above commutation relations it follows that

$$
\begin{aligned}
[A_i, A_j] &= i\epsilon_{ijk} A_k \\
[B_i, B_j] &= i\epsilon_{ijk} B_k \\
[A_i, B_j] &= 0
\end{aligned} \tag{3.3}
$$

The conclusion is that A and B satisfy two separate $SU(2)$ algebras and so $SO(3,1)$ is said to be locally isomorphic to $SU(2) \times SU(2)$. Since the representations of $SU(2)$ are labelled by $j = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$ those of the Lorentz group will receive a label (j_A, j_B).

The simplest nontrivial representation of all is $(\frac{1}{2}, 0)$, these are the Weyl spinors. In this case we must have:

$$
J_i = \tfrac{1}{2}\sigma_i \qquad \text{and} \qquad iK_i = \tfrac{1}{2}\sigma_i \tag{3.4}
$$

These relations lead us to the transformation rules of the Weyl spinors. Let a be a Weyl spinor, then we have

$$
\begin{aligned}
a &\rightarrow e^{-\frac{i}{2}\sigma\cdot\theta} a \qquad \text{(rotation)} \\
a &\rightarrow e^{-\frac{1}{2}\sigma\cdot\eta} a \qquad \text{(boost)}
\end{aligned} \tag{3.5}
$$

It is important to note that the rotation is represented by a unitary matrix and the boost by a hermitian one. The behavior under rotations shows that a Weyl spinor carries a spin of one half. From these two component Weyl spinors one can construct the four component Dirac spinor.

3.2 Dirac spinors

Consider two (possibly) different Weyl spinors a and b. These will form the building blocks of a Dirac spinor ψ:

$$\psi = \begin{pmatrix} a \\ \epsilon b^* \end{pmatrix} \qquad \text{with} \qquad \epsilon := i\sigma^2. \tag{3.6}$$

Some texts refer to a and the combination ϵb^* as left and right handed Weyl spinors respectively [5]. This will become clear in equation (3.8). If a transforms under the $(1/2, 0)$ representation then ϵa^* does so under the (0,1/2) representation, which differs by a minus sign in the exponent of the boost transformation [9] in (3.5). Suppose a transforms under *rotations* as $a \to Ua$, then, using $\sigma^2 U \sigma^2 = U^*$, we should find that ϵa^* transforms in the same way:

$$\epsilon a^* \to \epsilon (Ua)^* = U(\epsilon a^*)$$

And it does! For boosts, on the other hand, the transformation will be $a \to Ha$, H being hermitian and we hope to see a change in sign in the exponent.

$$\epsilon a^* \to \epsilon (Ha)^* = H^{-1}(\epsilon a^*)$$

Where the identity $\sigma^2 H^* \sigma^2 = H^{-1}$, which holds for 2×2 hermitian matrices with unit determinant, gave the last equality. Now look back at (3.5); the bottom two components of the Dirac spinor indeed need a minus sign in the exponent of the boost transformation. This verifies the statement of the combination ϵa^* being in the (0,1/2) representation of the Lorentz group. For obvious reasons people say that a Dirac spinor lives in the $(1/2, 0) \oplus (0, 1/2)$ representation of the Lorentz group.

Before we move on to write down the mass terms, there are two more things to be discussed, starting with the charge conjugation. The spinor conjugate to (3.6), ψ^c, is defined as follows:

$$\psi^c = \begin{pmatrix} b \\ \epsilon a^* \end{pmatrix} = \begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \psi^* \tag{3.7}$$

The first matrix is given the name C and the second is just γ^0, in the chiral basis[2], so in short we have $\psi^c = C\gamma^0\psi^*$.

In the special case that the two Weyl spinors constituting ψ are identical ($a = b$) we obviously have $\psi = \psi^c$. This is called a Majorana spinor.

In chapter 2 the projection operators were mentioned. Now we will restate the results in the 'new language' of Weyl spinors. When $P_\pm$ are written out explicitly using the gamma matrices in the chiral basis we see that we get when acting on ψ

[2]This is the basis used throughout this report. See appendix A for more details.

$$\begin{aligned}
\psi_L &= \tfrac{1}{2}(1-\gamma^5)\psi = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} a \\ \epsilon b^* \end{pmatrix} = \begin{pmatrix} a \\ 0 \end{pmatrix} \\
\psi_R &= \begin{pmatrix} 0 \\ \epsilon b^* \end{pmatrix}
\end{aligned} \tag{3.8}$$

So we see that that the projection operators pick out the upper or lower two components.

3.3 The Majorana mass term

This mass term is formed from a single Weyl spinor a and needs to be a Lorentz scalar:

$$\mathcal{L} = \tfrac{1}{2}m(a^T\epsilon a - a^\dagger \epsilon a^*) = \tfrac{1}{2}m(a^T\epsilon a + h.c.) \tag{3.9}$$

One can check the Lorentz invariance by using (3.5). One thing that can immediately be seen from (3.9), is that these terms are forbidden for particles carrying an unbroken $U(1)$ charge. For this reason charged particles can not have such a mass term. Note that the global $U(1)$ symmetry associated with lepton number conservation is broken by the Majorana mass term.

When doing algebraic manipulations involving Weyl spinors one should be aware of their Grassman nature, that is, their components are anticommuting numbers. Looking at the Majorana mass term one can see this is absolutely vital. Consider the following calculation (which is wrong!): $a^T\epsilon b = a_i\epsilon_{ij}b_j = b_j(-\epsilon_{ji})a_i = -b^T\epsilon a$. In the case of a Majorana mass term, this would clearly be very disturbing. Despite all this, when doing a Hermitian conjugation, there is no need to insert an extra minus sign for swapping the two spinors as can be seen in the second equality of equation (3.9).

It is possible to form a Majorana mass from a Dirac spinor. As can be seen from (3.9) only one of the two Weyl spinors plays a role. This can be accomplished by using only ψ_L:

$$\mathcal{L} = -\tfrac{1}{2}m(\psi_L^T\, C\psi_L + h.c.) \tag{3.10}$$

The C is inserted to produce the required ϵ and the fact that it needs a minus sign can be traced back to the definition of C. If ψ is as in (3.6), equation (3.10) contains only a's. It is easily checked that

$$\mathcal{L} = -\tfrac{1}{2}m(\psi_R^T\, C\psi_R + h.c.) \tag{3.11}$$

is a Majorana mass term for b.

There are other ways to arrive at a Majorana mass using either Dirac or Majorana spinors. Table 3.1 summarizes all the results which can be verified with relatively easy matrix manipulations.

3.4 The Dirac mass term

First we construct the mass term using Weyl spinors and then we give the more familiar form using their four component brothers, the Dirac spinor.

The Dirac mass can be written in terms of two Weyl spinors

$$\mathcal{L} = m(a_L^T \epsilon b - b^\dagger \epsilon a^*) \tag{3.12}$$

Its Lorentz invariance follows from that of the Majorana mass term: b transforms exactly as a does. If a and b carry a $U(1)$ charge they need to have opposite sign or, in other words, if a transforms as $a \to Ua$ then b should go like $b \to U^* b$.

Equation (3.12) can also be written in terms of a single Dirac spinor. This is of course the guise in which it is most often encountered:

$$\mathcal{L} = -m\bar{\psi}\psi = -m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L). \tag{3.13}$$

Note that, in contrast to the Majorana mass, the left is coupled to the right handed part.

Again, the mass term can be written in terms of all types of spinors we have met sofar. The table below summarizes this.

Spinor	Majorana mass	Dirac mass
Weyl	$\frac{1}{2}m(a^T \epsilon a + h.c.)$	$m(a_L^T \epsilon b + h.c.)$
Majorana	$-\frac{1}{2}m\overline{\psi}\psi$	
Dirac	$-\frac{1}{2}m(\psi_L^T C \psi_L + h.c)$	$-m((\psi^c)_L^T C \psi_L + h.c.)$
Dirac		$-m\overline{\psi}\psi$

Table 3.1: Overview of the two types of mass terms using different kinds of spinors.

CHAPTER 4

Neutrino masses

In the last two chapters we have seen how neutrinos are accommodated in the standard model and also what kind of fermion mass terms there are. In the present chapter we see how a general mass term can be written neatly in a matrix form and that the massive neutrinos can lead to neutrino oscillations, a phenomenon alien to the standard model.

But, first a very short review will be given of the kind of experiments being done around the globe in an attempt inquire more and more accurate data on neutrino masses.

4.1 Experiments

In 1932 Enrico Fermi first theoretically introduced the neutrino in a description of β-decay. Some 25 years later the $\bar{\nu}_e$ was actually detected by Frederick Reines and Clyde Cowan [10]. The first experiment that, in retrospect, hinted towards the occurrence of neutrino oscillations was performed by Ray Davis using a chlorine based detector built in the 1960's, in an attempt to measure the neutrino flux from the sun. The observed flux was roughly one third of what was to be expected from the solar models. The *solar neutrino problem* was born.

One way to fix this problem was to adjust the solar model. Another solution speculated about massive neutrinos. If the weak eigenstates produced in interactions are in fact an admixture of non-degenerate mass eigenstates, oscillations between different types of neutrinos will occur which may explain

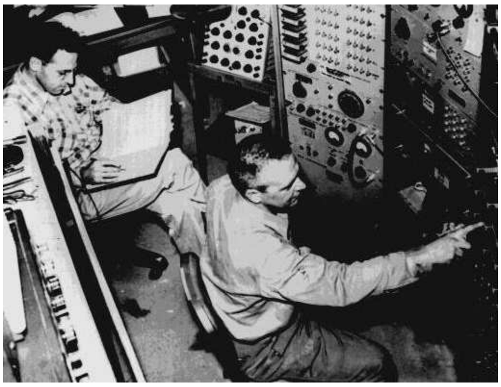

Figure 4.1: Frederick Reines in the control room.

the deficit found in Davis's experiment. Pontecorvo raised this possibility as early as 1969.

The strongest indication of oscillations came from the Superkamiokande experiment in Japan. The possible oscillations of atmospheric rather than solar neutrinos are measured. In this experiment an immensely big tank with 50,000 tons of pure water together with 11,200 photo multiplier tubes serve as a detector. Because in each detection event the direction of the neutrino can be resolved it is possible to distinguish between neutrino's coming from the sky or through the earth from below. The latter type, once detected, has had a fair bit of travelling so that oscillations become apparent in an asymmetry in the yield of the upward and downward travelling neutrinos. The Superkamiokande started acquiring data on April 1, 1996 which lead to their famous publication in 1998 [11] announcing the conclusion that the discrepancy just mentioned can perfectly be explained by neutrino oscillations. The inferred mixing angle between τ and μ neutrinos should have a value of $\sin^2 2\theta > 0.82$ and a mass squared difference[1] of $5 \times 10^{-4} < \Delta m^2 < 6 \times 10^{-3}$ eV^2. Experiments that study the oscillation of neutrinos can only give us a clue of the mass squared differences. No absolute scale can be found.

There is another class of experiment in which the β-decay of tritium is studied. As a product of the decay the neutrino, as well as the electron, carries away some energy. If the neutrino has zero rest mass, the maximum energy to be carried away by the electron is just the total energy produced in a single decay reaction. This picture changes in the case of massive neutrinos. In short, the high end of the energy spectrum of the electron provides information on the mass of the neutrino in an absolute sense. The most

[1]This means $\Delta(m^2)$, not $(\Delta m)^2$. Usually people write Δm^2

recent attempt of such an experiment is made by *The Mainz Neutrino Mass Experiment* group from the University of Mainz. The newest results were published in The European Physical Journal in 2005 [12]. It was found that $m^2(\nu_e) = (-0.6 \pm 2.2_{\text{stat}} \pm 2.1_{\text{syst}})$ eV^2/c^4. The results, curious as they may seem, were useful to derive an upper limit of $m(\nu_e) \leq 2.2$ eV/c^2 at a 95% confidence level.

At the moment in Karlsruhe the Katrin experiment is being prepared. It is an experiment of the same kind as the one in Mainz but the claim is that after three years of acquiring data the limit on $m(\nu_e)$ can be pushed down to 0.2 eV/c^2 (see for example: [13]), if, of course, the electron neutrino is indeed that light.

Besides experiments to determine the mass scale and mass differences, there are ways to determine wether the neutrino has a Majorana mass or not. An intriguing consequence of the presence of a Majorana mass term is that a process called neutrinoless double beta decay can take place which violates lepton number conservation by two units. Two neutrons decay each into a proton and an electron. The special thing is that the neutrinos remain virtual particles on an internal line (see diagram 4.2). The occurrence of neutrinoless double beta decay is not an indisputable proof of the existence of a Majorana mass term as there is the possibility of other, heavy particles causing this to happen. In the model discussed in chapter 5 the doubly charged component of the heavy triplet scalar field is a candidate. What may happen, is that the two W bosons from figure 4.2 merge into this heavy scalar, which in turn decays into two electrons. In chapter 6 we will make an estimate of the relative amplitude of these two processes.

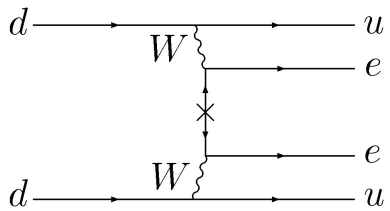

Figure 4.2: Feynman diagram for neutrinoless double β-decay.

4.2 Mass Terms for the Neutrino; the Seesaw Mechanism

The previous chapter may have left the reader with the impression that a field has either a Dirac or a Majorana mass. In the most general situation, however, there is a combination of both. Since we do not really know which is the case for the neutrino, it is useful to look at the general situation and of course to a particularly interesting special case, known as the seesaw scenario. In the SM people found that there was no need for a RH neutrino.

For the time being we just assume it is there, in order to have the possibility of a Dirac mass. Also, for now, *we will not bother with the question as to what model and what scalar field content could give rise to the situation described here.* That we will postpone to chapter 5.

4.2.1 One Generation

Given the left and right handed parts of the neutrino field, ν_L and ν_R respectively, one may write down the general mass term using Dirac spinors:

$$\begin{aligned}\mathcal{L}_{mass,\nu} = & - & m(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L) - \frac{1}{2}M_L(\nu_L^T C\nu_L - \nu_L^\dagger C\nu_L^*) \\ & - & \frac{1}{2}M_R(\nu_R^T C\nu_R - \nu_R^\dagger C\nu_R^*).\end{aligned} \tag{4.1}$$

Where all the "+h.c.'s" are written out in full. Now define $\nu := \nu_L$ and $N := (\nu_R)^c$ to arrive at the matrix expression. The third line in the table in the Dirac mass column, contains the object $(\psi^c)_L$. What does this mean? The conjugation interchanges the two Weyl spinors and the left projection gets rid of the lower two components. Obviously, one can just as well throw away the upper two and then do the conjugation: $(\psi^c)_L = (\psi_R)^c$. With this in mind we can use the third line of the table to rewrite the Dirac term in (4.1) using ν and N only. Some similar work can be done with the Majorana terms in (4.1) to find eventually

$$\mathcal{L} = -\frac{1}{2}\begin{pmatrix} \nu^T & N^T \end{pmatrix}\begin{pmatrix} M_L & m \\ m & M_R \end{pmatrix}\begin{pmatrix} C\nu \\ CN \end{pmatrix} + h.c. \tag{4.2}$$

For the last two equations to be equal we must have that $\nu^T CN = N^T C\nu$. Writing it out with the (spinor) indices explicit this seems to work out: $\nu^T CN = C_{ij}\nu_i N_j = -(C^T)_{ji}N_j\nu_j = -(-C)_{ji}N_j\nu_j = N^T C\nu$. The two minus signs arising from the Grassmann nature of the numbers and of C being anti symmetric cancel out.

The next thing one ought to do is find the eigenvalues of this matrix. They are given by

$$M_{1,2} = \frac{1}{2}[M_R + M_L \pm \sqrt{(M_R - M_L)^2 + 4m^2}]. \tag{4.3}$$

Let us look at a special limiting case. A common scheme is the so-called *seesaw mechanism*, which was first introduced by R. N. Mohapatra and G. Senjanovic [15]. What we ultimately want is to have a light neutrino. This can be accomplished by making M_R much larger than both m and M_L. In this case ($M_R \gg m, M_L$) eigenvalues of the mass matrix in this case are approximated by

$$\begin{aligned} M_\nu &\simeq M_L - \frac{m^2}{M_R} \\ M_N &\simeq M_R. \end{aligned} \tag{4.4}$$

Since the off-diagonal elements are small compared to M_L and M_R one can still speak of a 'mainly ν' and 'mainly N' neutrino; the mixing is just minute. Hence the labelling in (4.4). The large M_R will naturally be at the scale of the new physics, which also happens in the model to be seen in the next chapter.

4.2.2 Three Generations

Sofar the discussion was focussed on the case of one flavour (or 'generation'). As far as we know now their are three flavours in nature. It has actually been confirmed in an experiment performed at CERN [14], in which the decay width of the Z boson was studied, that there are indeed three types of *light* neutrinos.

The discussion of the 'one generation' section can be straightforwardly extended the three generation case. The mass term in equation (4.1) or equivalently (4.2) is duplicated for the other two generations. This can again be written compactly in matrix form

$$\mathcal{L} = -\frac{1}{2} \begin{pmatrix} \nu^T & N^T \end{pmatrix} \begin{pmatrix} M_L & m \\ m^T & M_R \end{pmatrix} \begin{pmatrix} C\nu \\ CN \end{pmatrix} + h.c. \tag{4.5}$$

This time the objects ν and N are vectors of spinors:

$$\nu = (\nu_e, \nu_\mu, \nu_\tau)^T; \qquad N = (N_e, N_\mu, N_\tau)^T.$$

Assuming the entries of M_R are much larger than those of the other submatrices, we can, following reference [16], bring the neutrino mass matrix in block diagonal form by quite literally rotating away the off-diagonal 'elements' m from (4.5). Since the matrix is very close to wanted form, a small rotation suffices:

$$\begin{aligned} \begin{pmatrix} M_\nu & 0 \\ 0 & M_N \end{pmatrix} &= \begin{pmatrix} 1 & \epsilon \\ -\epsilon & 1 \end{pmatrix}^T \begin{pmatrix} M_L & m \\ m^T & M_R \end{pmatrix} \begin{pmatrix} 1 & \epsilon \\ -\epsilon & 1 \end{pmatrix} \\ &\approx \begin{pmatrix} M_L - mM_R^{-1}m^T & 0 \\ 0 & M_R \end{pmatrix}, \end{aligned} \tag{4.6}$$

ϵ being equal to $M_R^{-1}m^T$. The blocks of zeroes are not identically zero but the ratio of its entries to those on the diagonal blocks is of order ϵ. The upper left corner shows the famous see saw formula for the light neutrinos and is clearly the analog of formula (4.4). We will assume the diagonal blocks to be symmetric, which is true for the model we will restrict our attention to in this report, the left-right symmetric model.

If we call the original mass matrix from (4.5) $\mathcal{M}$, the block diagonal form $\mathcal{M}_{BD}$ and rotation matrix in the previous equation S, we can write $\mathcal{M} = S\mathcal{M}_{BD}S^T$. The two symmetric blocks occurring in $\mathcal{M}_{BD}$ can be further transformed into a diagonal form $M^d_{\nu,N} = V^T_{\nu,N}M_{\nu,N}V_{\nu,N}$ where the V's are 3×3 unitary matrices. If we define $V := \begin{pmatrix} V_\nu & 0 \\ 0 & V_N \end{pmatrix}$ We can write the original matrix $\mathcal{M}$ in terms of its diagonal form $\mathcal{M}_D$ in the following way: $\mathcal{M} = SV^*\mathcal{M}_DV^\dagger S^T$. This implies that

$$\mathcal{N}_{Gauge} = SV\mathcal{N}_{Mass} \qquad \text{with} \qquad \mathcal{N}_{(...)} = \begin{pmatrix} \nu_{(...)} \\ N_{(...)} \end{pmatrix}. \tag{4.7}$$

Usually the effects of the matrix S are ignored, which is to say that the mixing between the light left handed and the heavy right handed neutrinos is extremely small (for example [16, 20])

The fact that the mass eigenstates are essentially different from the gauge eigenstates, is in great contrast with the situation in the standard model. There we saw that the neutrino fields can be redefined, to 'absorb' the effects of the mass eigenstates of the *charged* leptons being different from the gauge eigenstates. Consequently in the SM lepton flavour is conserved and the charged currents are diagonal in the flavours.

Now that we know the neutrinos have mass and relations like (4.7) seem to hold, such harmless redefinitions are no longer possible. Consider a left handed charged current interaction term. Besides the neutrinos the charged leptons need a redefinition. To distinguish this matrix from the one in equation (4.7), both are given an index. In terms of the *mass* eigenstates the charged current interaction is no longer diagonal. Up to some factors we have

$$\mathcal{L}^{cc} = W^+_\mu \bar{l}^i (V_l^\dagger V_\nu)^{ij} \gamma^\mu \nu^j, \tag{4.8}$$

with the flavour indices explicitly showed and where $l = (e, \mu, \tau)^T$ and also ν is a 3-vector. The matrix between brackets is given name of its own: the NMS-matrix, after Maki, Nakagawa and Sakata[2]:

$$U^{NMS} = V_l^\dagger V_\nu. \tag{4.9}$$

there is no reason for this matrix to be diagonal so there are vertices with a charged gauge boson, a charged lepton and neutrino of possibly different flavour. This lies at the heart of the oscillation phenomena mentioned in earlier in the short summary of experiments on neutrinos.

One can show that field redefinitions can be made with the purpose of reducing the number of parameters in the NMS matrix by removing some of entry's complex phases. In general, the number of phases phases cannot be reduced further than three, one of which plays the same role as the CP-violating phase in the CKM-matrix occurring analogously in the quark sector. The other two are the so-called Majorana phases ($\alpha_{1,2}$ below). A common parametrization of the NMS-matrix is (cf. [16]).

$$\begin{aligned} U^{NMS} &= R_{23} R_{13}^\delta R_{23} \times \mathrm{diag}(1, e^{i\alpha_1}, e^{i\alpha_2}) \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \\ &\quad \times \mathrm{diag}(1, e^{i\alpha_1}, e^{i\alpha_2}). \end{aligned} \tag{4.10}$$

The notation

$$R_{13}^\delta = \begin{pmatrix} c_{13} & s_{13}e^{-i\delta} & 0 \\ -s_{13}e^{i\delta} & c_{13} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is used. The matrices R_{12} (R_{23}) represent similar rotations, but in the 1-2 (2-3) plane with $\delta = 0$. Obviously c_{12} stands for $\cos(\theta_{12})$, etc.

[2]Sometimes called the PNMS matrix; the P standing for Pontecorvo.

4.3 Minimal Extensions of the Standard Model

In the previous sections the Seesaw mechanism was discussed. The simplest way to implement this in the standard model is to introduce a right handed neutrino field, in the literature often denoted by N_R. We need a coupling similar to (2.13) involving the left and right handed neutrino fields to produce the light Dirac mass needed in the seesaw scenario:

$$\mathcal{L}_Y = -f_\nu^{ij}\bar{L}^i\tilde{\phi}N_R^j. \tag{4.11}$$

Here $\tilde{\phi} := i\sigma_2\phi^*$ transforms exactly like ϕ; it is also an $SU(2)$ doublet. This construction is needed to involve the upper component of L, the left handed neutrino and is also used to give the up quarks mass. Looking at the above Yukawa term, one sees that N_R is obviously a singlet under $SU(2)$. It does not carry a $U(1)_Y$ charge since $Y(L) = -1$ and $Y(\phi) = 1$. We conclude it is a singlet under the standard model gauge group. Therefore, a Majorana mass term for the new field is allowed:

$$\mathcal{L} = -\tfrac{1}{2}N_R^{iT}M_R^{ij}CN_R^j. \tag{4.12}$$

When ϕ acquires its vacuum expectation value, the above two terms result in the mass matrix

$$\frac{1}{2}\begin{pmatrix} 0 & vf \\ vf^T & M_R \end{pmatrix}. \tag{4.13}$$

This is often referred to as a type I seesaw mechanism, and is responsible for the second term in the mass M_ν in equation (4.6).

The scenario of the singlet right handed neutrinos cannot give rise to Majorana mass for the left handed neutrino. This can be achieved in a different way. Suppose that in the standard model, besides the familiar Higgs doublet, there is an extra set of scalar fields: an $SU(2)$ triplet field which would be a 2×2 matrix parametrized as $\Delta = \delta\cdot\sigma$. It would transform by the rule $\Delta \to U\Delta U^\dagger$, so that a Yukawa coupling of the kind

$$L^T\epsilon\Delta CL \tag{4.14}$$

is allowed by the local symmetry. That is, if we assign the hypercharge $Y(\Delta) = +2$. By choosing the suitable component to acquire a vacuum expectation value one can assure that a Majorana mass term arises only for the neutrino, not for the electron. One may wonder if this implies extra Yukawa terms for the quarks as well. The hypercharge of the quark doublet is $\frac{2}{3}$ so that no such couplings are allowed in the quark sector. The field Δ can acquire a vev to create a mass term $m\nu^TC\nu$ via the above Yukawa coupling. This is called a Type II contribution and corresponds to the first term in (4.6).

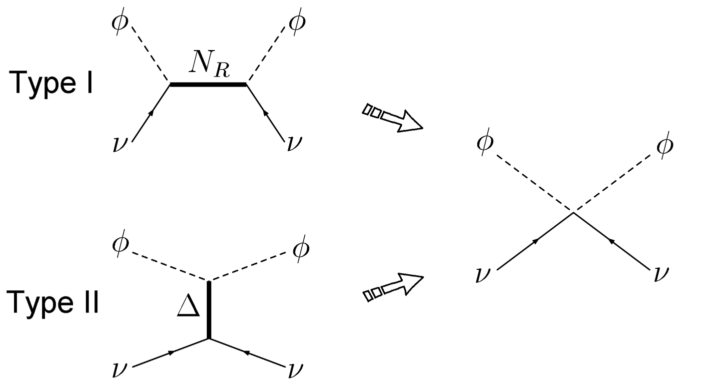

Figure 4.3: At energies far below the masses of the heavy particles, both the type I and II seesaw mechanism effectively lead to a dimension 5 operator involving two scalar fields and two fermions.

The problem is that if the new particles involved (N_R and Δ) are heavy, which should be the case for the Yukawa couplings not to be too small, there is no way of distinguishing between the two scenarios. Effectively, they both give rise to an effective operator schematically denoted by $\mathcal{L}_5 = GL\phi\phi L$. Figure 4.3 shows how at energies much lower than the intermediate particle masses one effectively appears to have a term in the Lagrangian like $\mathcal{L}_5$. The intermediate particle stays on an internal line since it is too heavy to actually be produced so that 'from the outside' it simply looks like a four particle point interaction.

This kind of reasoning lies at the heart of what is called Effective Field Theory (short introduction in [6], chapter III.2, or [17]), usually abbreviated by EFT. Of a given theory with both 'heavy' and 'light' particles one can produce its low energy limit. In doing so, one writes the theory in terms of the light particles only. The standard example is the Fermi theory of weak interactions where the heavy W and Z bosons are the heavy particles, invisible at energies way below some tens of GeV's. The Lagrangian contains interaction terms with four fermion fields. It is very similar to the situation in figure 4.3. The effective field theory can approximate the full theory with arbitrary precision by adding more terms to the Lagrangian, which becomes a series expansion in momenta over mass or, equivalently, there will be an infinite number of derivative corrections.

The subscript 5 in $\mathcal{L}_5$ is to say that the mass dimension of the combination[3] $[\psi\phi\phi\psi] = 5$ is five so that [G]=-1. A non-renormalizable operator like this was for a long time considered unacceptable. Suppose one wants to calculate an amplitude involving this interaction. It can be argued that terms of the order G^n behave like $G^n E^{n-1}$ (see for example [6]) where E is the center of mass energy of the process. Without bothering about the coefficients, the amplitude can be written as $G(1 + GE + (GE)^3 + ...)$. At energies comparable to G^{-1} all terms become equally important and things go wrong. Therefore, the theory must have a cut off, which in modern views is not an indication the theory is wrong, but that it is only a low energy effective description of a bigger scenario involving heavier particles at the scale of the cut off. In the above notation this means these heavy particles have masses of the order G^{-1}. In the 'bottom-down' approach of figure 4.3 it probably makes more sense to say that G^{-1} must be of the order of the masses of the heavy particles.

The point to be made is that both attempts to extend the standard model are not very fruitful approaches since it gives no clue whatsoever about the scale of the new physics. The masses of the particles are not linked to those of other particles. In the next chapter we will discuss a model in which essentially the above two methods are implemented. There are indeed right handed neutrinos in there. They are singlets under the standard model group, which is a sub group off the full gauge group. Also, there are $SU(2)$ triplets allowing for Yukawa couplings similar to the one in equation (4.14).

[3] Square brackets around some object denote its mass dimension.

CHAPTER 5

The Left-Right Symmetric Model

Ever since the birth of the standard model physicists have been trying many alternatives theories with more complex gauge groups. Famous examples are the SO(10) and SU(5) Grand Unified Theories that try to make the values of the different coupling constants we know from QED, the weak and strong interaction to converge to a single value at some very high energy.

Our goals are somewhat more modest and no renormalization group flow will be needed. The aim is to give the neutrino a small mass in the context of a model that very closely resembles the standard model at low energies, that is, energies at which present day experiments take place. Our choice is to focus on a so-called left-right symmetric model which seems to be one of the most straightforward extensions of the standard model. Its gauge group is $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ which is, with its 7 generators and as many gauge bosons, obviously bigger than the SM group. It provides a description of the weak interactions only.

The main idea is that the essential difference between the left and right handed (from now on also LH and RH, respectively) parts of the fermion fields, as present in the SM, is abandoned. In other words, parity is an explicit symmetry until spontaneous symmetry breaking takes place. At the end of section 5.1 we will see how this discrete symmetry is implemented in the model. Because of the pronounced V-A structure[1] of the weak interactions, we know parity *must* be broken at some point.

[1]This is another way of saying that it only terms like $\bar{\psi}\gamma^{\mu}(1-\gamma^5)\psi$ occur in the interactions. This, in turn, implies that only the left handed parts of the fields play a role.

Another peculiar feature of the model is the gauged $U(1)$ B-L symmetry. The difference between baryon and lepton number is a conserved quantity but, again, only until spontaneous symmetry breaking takes place. Actually, because there are no couplings between quarks and leptons in the theory, L and B will be conserved separately. We have seen in section 3.3 that a Majorana mass term, which we want to appear upon symmetry breaking, violates lepton number conservation. Some authors say that the B-L makes more physical sense then the hyper charge from the standard model, which makes it a more attractive choice. The fact that the $B-L$ is anomaly free, is sometimes used as an answer to the question as to why one should gauge the symmetry. Another reason is that $B-L$ automatically appears as a gauged symmetry in $SO(10)$ grand unification [18]. Since the left-right symmetric model can be embedded in $SO(10)$, one is in some sense looking at a low energy limit of this GUT.

The left-right symmetric models were first introduced around 1974 by Pati and Salam [19] but also Rabindra N. Mohapatra and Goran Senjanovic were very active in this field. In 1981 the latter two gentlemen wrote an article with the title *Neutrino masses in gauge models with spontaneous parity violation* [20]. That article forms the backbone of the present chapter.

5.1 Matter and Higgs Fields

Fermion fields

The Quark fields are arranged in left handed (LH) doublets of $SU(2)_L$, just as in the standard model, and right handed (RH) doublets of $SU(2)_R$:

$$Q^i_R = \begin{pmatrix} u^i_R \\ d^i_R \end{pmatrix}. \tag{5.1}$$

where i is the generation-index so that $u^i = u, c, t$ and $d^i = d, s, b$. In the leptonic sector we have the same: besides the familiar LH lepton doublet, L_L, there is the RH version

$$L^i_R = \begin{pmatrix} \nu^i_R \\ l^i_R \end{pmatrix}. \tag{5.2}$$

with $\nu^i = \nu_e, \nu_\mu, \nu_\tau$ and $l^i = e, \mu, \tau$. This means that there is room for the right handed neutrino. The action of a group element of $SU(2)_L \otimes SU(2)_R$ on these doublets is as follows[2]:

[2]The index $\mathcal{H}$ is a 'left-right' index valued either L or R.

$$\begin{array}{rcl} Q_{\mathcal{H}} & \rightarrow & U_{\mathcal{H}} Q_{\mathcal{H}} \\ L_{\mathcal{H}} & \rightarrow & U_{\mathcal{H}} L_{\mathcal{H}} \end{array}$$

Where the index $\mathcal{H}$ is a 'left-right' index valued either L or R. The matrix $U_{\mathcal{H}}$ is any local $SU(2)_{\mathcal{H}}$ transformation. The quarks have $q_{B-L} = 1/3$ and the leptons -1, so that under $U(1)_{B-L}$ the doublets transform as

$$\begin{array}{rcl} Q_{\mathcal{H}} & \rightarrow & e^{iq_{B-L}\alpha(x)} Q_{\mathcal{H}} = e^{i\alpha(x)/3} Q_{\mathcal{H}} \\ L_{\mathcal{H}} & \rightarrow & e^{iq_{B-L}\alpha(x)} L_{\mathcal{H}} = e^{-i\alpha(x)} L_{\mathcal{H}}. \end{array} \quad (5.3)$$

The minus sign in the exponent is a convention chosen here. There is some freedom here, as long as the transformation rule of the fermion fields, the gauge fields and the precise form of the covariant derivative are consistent with each other (more on that in appendix B). In summary we have $L_L(2, 0, -1)$, $L_R(0, 2, -1)$, $Q_L(2, 0, 1/3)$ and $Q_R(0, 2, -1/3)$,

Before we move on to the Higgs fields, a short remark is in place. It needs to be stressed that the field ν_R needs not to pair up with its LH partner to a Dirac spinor. In section 3.4 we have seen that two Weyl spinors can be combined into a Dirac spinor if they both have zero Majorana mass. Here, the option for Majorana masses is left open.

Scalar fields and masses

What kind of scalar fields do we need? To produce Dirac masses the LH and RH spinors should be brought together by a Yukawa coupling of L_L to L_R and some scalar field. The obvious choice would be a 2×2 matrix field Φ transforming like

$$\Phi \rightarrow U_L \Phi U_R^\dagger, \quad (5.4)$$

called a bi-doublet, allowing the following gauge invariant couplings

$$\begin{array}{rcl} \mathcal{L}_Y & = & f_1^{ij} \overline{L}_L^i \Phi L_R^j + f_2^{ij} \overline{L}_L^i \tilde{\Phi} L_R^j \\ & + & F_1^{ij} \overline{Q}_L^i \Phi Q_R^j + F_2^{ij} \overline{Q}_L^i \tilde{\Phi} Q_R^j \quad + \quad \text{h.c.} \end{array} \quad (5.5)$$

We use the notation $\tilde{\Phi} := \sigma^2 \Phi^* \sigma^2$. The important thing is that $\tilde{\Phi}$ transforms in the same way Φ does. It is clear that q_{B-L} for this field is zero.

So far so good, but the couplings just mentioned will never produce a Majorana mass. Recall from table 3.4 that we need a term like $m\psi_L^T C\psi_L$. To this end a field $\Delta_{\mathcal{H}}$ is introduced allowing for the following Yukawa couplings

$$\mathcal{L}_Y = h^{ij}(L_L^{iT} C\epsilon\Delta_L L_L^j + L_R^{iT} C\epsilon\Delta_R L_R^j). \tag{5.6}$$

Note that both terms have the same 'coupling strength' h^{ij}. This is to preserve the discrete left-right symmetry. The ϵ makes sure that the transformation of Δ_L

$$\Delta_{\mathcal{H}} \to U_{\mathcal{H}}\Delta_{\mathcal{H}}U_{\mathcal{H}}^\dagger, \tag{5.7}$$

leaves the Yukawa term untouched. This can be seen by applying the rule $\epsilon U = U^*\epsilon$. Since none of the lepton fields in (5.6) has a complex conjugation, Δ has $q_{B-L} = +2$. This charge assignment forbids a coupling of quarks to $\Delta_{\mathcal{H}}$

In summary we have the scalar fields $\Phi(2, 2^*, 0)$, $\Delta_L(3, 0, +2)$ and $\Delta_R(0, 3, +2)$. The precise parametrization of the matrix fields will be dealt with in the next section.

Parity symmetry

The Lagrangian that appeared in bits and pieces in this section contains many parameters. The parity symmetry will put some restrictions on them. One of them we have already encountered: the Majorana couplings of the left and right handed fields must be the same. The obvious way to impose the discrete parity symmetry is to demand invariance under:

$$\begin{aligned} Q_L^i, L_L^i &\leftrightarrow Q_R^i, L_R^i \\ \Phi &\to \Phi^\dagger. \end{aligned} \tag{5.8}$$

The most obvious consequence is that the gauge couplings of the left and right handed $SU(2)$ are equal ($g_L = g_R =: g$). Looking at the Dirac Yukawa couplings (5.5) it can be seen that under (5.8) each term must transform into its hermitian conjugate. Therefore, the matrices $f_{1,2}$ and $F_{1,2}$ should be taken Hermitian.

5.2 Kinetic Terms of the Lepton and Higgs fields; Gauge Fields

Sofar we have not looked at the kinetic terms of the fields. To keep them gauge invariant a covariant derivative is needed. So first the covariant derivatives will be given, and then the kinetic terms.

It looks very similar to the SM case, but there are some interesting modifications. The full list reads[3]:

$$\begin{aligned}
D_\mu L_{\mathcal{H}} &= (\partial_\mu - igA^{\mathcal{H}}_\mu + \frac{ig'}{2}B_\mu)L_{\mathcal{H}} \\
D_\mu Q_{\mathcal{H}} &= (\partial_\mu - igA^{\mathcal{H}}_\mu - \frac{ig'}{6}B_\mu)Q_{\mathcal{H}} \\
D_\mu \Phi &= \partial_\mu \Phi - ig(A^L_\mu \Phi - \Phi A^R_\mu) \\
D_\mu \Delta_{\mathcal{H}} &= (\partial_\mu - ig'B_\mu)\Delta_{\mathcal{H}} - ig[A^{\mathcal{H}}_\mu, \Delta_{\mathcal{H}}]
\end{aligned} \tag{5.9}$$

using the notation $A^L_\mu := \tau^a A^{L,a}_\mu$ which implies a grand total of 7 gauge fields. Especially the last three lines may look somewhat unfamiliar: there are terms occurring like $ig\Phi A^R_\mu$ where the gauge fields are placed to the right. This has everything to do with the way Φ transforms. The following set of transformations makes the complete framework consistent:

$$\begin{aligned}
A^{a,L}_\mu &\rightarrow U_L A^{a,L}_\mu U_L^\dagger - \frac{i}{g}(\partial_\mu U_L)U_L^\dagger \\
A^{a,R}_\mu &\rightarrow U_R A^{a,R}_\mu U_R^\dagger - \frac{i}{g}(\partial_\mu U_R)U_R^\dagger \\
B_\mu &\rightarrow B_\mu - \frac{1}{g'}\partial_\mu \alpha.
\end{aligned} \tag{5.10}$$

Details on the previous two groups of equations can be found in appendix B.

As for the kinetic terms of the lepton doublets: they are exactly the same as in the SM:

$$i\overline{L}_{\mathcal{H}}\gamma^\mu D_\mu L_{\mathcal{H}}. \tag{5.11}$$

More interesting is the case of the Higgs fields. They are scalar fields grouped in matrices. The fields Φ en $\Delta_{\mathcal{H}}$ are parametrized as follows [20]:

[3] To arrive at the factor in front of B_μ, the q in the last boxed equation of appendix B should be divided by 2. This is (again...) purely conventional.

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \qquad \tilde{\Phi} = \begin{pmatrix} \phi_2^{0*} & -\phi_2^- \\ -\phi_1^+ & \phi_1^{0*} \end{pmatrix} \tag{5.12}$$

$$\Delta_{\mathcal{H}} = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta^+ & \delta^{++} \\ \delta^0 & -\frac{1}{\sqrt{2}}\delta^+ \end{pmatrix}_{\mathcal{H}} \tag{5.13}$$

Before we proceed with the kinetic terms of these fields we note the following. Let A_{ij} be a square matrix. Then $\mathrm{tr}(A^\dagger A) = (A^\dagger)_{ij}A_{ji} = (A^*)_{ji}A_{ji}$. In words: the outcome is the sum of the absolute value squared of all matrix elements. With this in mind and the above definitions of Φ and $\Delta_{\mathcal{H}}$ it follows that if we take

$$\mathcal{L}_{kin,Higgs} = \mathrm{tr}\Big[(D_\mu\Delta_L)^\dagger(D^\mu\Delta_L)+(D_\mu\Delta_R)^\dagger(D^\mu\Delta_R)+(D_\mu\Phi)^\dagger(D^\mu\Phi)\Big] \tag{5.14}$$

we get the 'standard' *complex* scalar field kinetic term (ie. $(\partial_\mu\phi)^\dagger(\partial^\mu\phi)$) for the separate components of the matrix fields plus of course some interaction terms between Higgs fields and the various gauge fields. Using the cyclic property of the trace operation, it is easily checked that these are indeed gauge invariant. Look for example at the term involving Φ:

$$\mathrm{tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] \to \mathrm{tr}[U_R(D_\mu\Phi)^\dagger U_L^\dagger U_L(D^\mu\Phi)U_R^\dagger] \tag{5.15}$$

where the different U_L and U_R matrices all combine to unit matrices.

5.3 Symmetry Breaking Pattern in the Model; Gauge Boson Masses

When people discuss models to describe the weak interactions with bigger gauge groups than the SM group, one usually constructs the Higgs potential in such a way that there is more than one breaking scale. In the present model there are two stages of symmetry breaking, in contrast with just one in the standard model. In the discussion below we will concentrate on the gauge bosons only. The implications of the breaking on the leptons will be discussed later in section 5.5.

5.3.1 First Stage: $G_{LR} \to G_{SM}$

In the first stage (that is, occurring at the higher of the energies) Δ_R acquires the real[4] vacuum expectation value

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}. \tag{5.16}$$

This breaks the $SU(2)_R$ symmetry by giving the RH neutrino a Majorana mass via the Yukawa coupling (5.6). Also, the gauge fields of $SU(2)_R$ become massive through the kinetic terms of Δ_R. To keep the expressions somewhat more compact, define $K := \langle \Delta_R \rangle$. The terms from (5.14) that are relevant to the mass of the RH gauge fields are:

$$\begin{aligned} \mathcal{L} &= \mathrm{tr}(ig' B_\mu K^\dagger + ig[K^\dagger, A_\mu^R])(-ig' B_\mu K - ig[A_\mu^R, K]) \\ &= -\frac{v_R^2}{2} \mathcal{A}_\mu^\dagger \begin{pmatrix} g^2 & & & \\ & g^2 & & \\ & & 2g^2 & -2gg' \\ & & -2gg' & 2g'^2 \end{pmatrix} \mathcal{A}^\mu. \end{aligned} \tag{5.17}$$

With $\mathcal{A}_\mu^\dagger := (A_\mu^{R,1}, A_\mu^{R,2}, A_\mu^{R,3}, B_\mu)$. The same procedure as in section 2.2 results in the following list of massive combinations:

Eigenvector	Mass	Field combination	New name
$(1, i, 0, 0)$	gv_R	$\frac{1}{\sqrt{2}}(A_\mu^{R,1} + iA_\mu^{R,2})$	$W_{R,\mu}^-$
$(1, -i, 0, 0)$	gv_R	$\frac{1}{\sqrt{2}}(A_\mu^{R,1} - iA_\mu^{R,2})$	$W_{R,\mu}^+$
$(0, 0, g, -g')$	$\sqrt{2(g^2 + g'^2)}v_R$	$\frac{1}{\sqrt{g^2+g'^2}}(gA_\mu^{R,3} - g'B_\mu)$	$Z_{R,\mu}$
$(0, 0, g', g)$	0	$\frac{1}{\sqrt{g^2+g'^2}}(g'A_\mu^{R,3} + gB_\mu)$	B'_μ

Table 5.1: The massive combinations after the first stage of symmetry breaking. All the $SU(2)_L$ gauge bosons remain massless after the this stage.

Now we will explicitly calculate $\bar{L}_R i\gamma^\mu D_\mu L_R$ in terms of the new gauge fields. First we note that the "Field combination" column of the above table can be summarized by:

[4]A lot can be said about choosing real or general complex VEV's of the potential. A little more on this can be found in appendix C.

$$\begin{pmatrix} W_R^- \\ W_R^+ \\ Z_R \\ B' \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} & & \\ 1/\sqrt{2} & -i/\sqrt{2} & & \\ & & \cos\alpha & -\sin\alpha \\ & & \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} A^{R,1} \\ A^{R,2} \\ A^{R,3} \\ B \end{pmatrix} \tag{5.18}$$

With $\cos\alpha := \frac{g}{\sqrt{g^2+g'^2}}$ and $\sin\alpha := \frac{g'}{\sqrt{g^2+g'^2}}$. The unitary matrix in front of $\mathcal{A}$ will be called S. If furthermore the notation

$$\mathcal{W}_\mu^T = (W_R^-, W_R^+, Z_R, B')_\mu,$$

is used, we can write $\mathcal{A} = S^\dagger \mathcal{W}$. Finally, a last piece shorthand notation: $P = i(g\vec{\tau}, g'\mathbf{1}/2)$ that is used to write[5] $D_\mu L_R$ as follows in terms of the new fields

$$\begin{aligned} D_\mu L_R &= (\partial_\mu - P\mathcal{A})L_R \\ &= (\partial_\mu - PS^\dagger \mathcal{W})L_R \\ &= \Big[\partial_\mu - i\frac{g}{\sqrt{2}}(W_{R,\mu}^+\tau^+ + W_{R,\mu}^-\tau^-) - i\frac{1}{\sqrt{g^2+g'^2}} Z_{R,\mu}(g^2\tau^3 + g'^2\frac{\mathbf{1}}{2}) \\ &- i\frac{gg'}{\sqrt{g^2+g'^2}} B'_\mu(\tau^3 - \frac{\mathbf{1}}{2})\Big] L_R, \end{aligned} \tag{5.19}$$

using the notation $\tau^\pm = \tau^1 \pm i\tau^2$. The effect of τ^+ (τ^-) on L_R is that it annihilates its lower (upper) component. Note that in the last term the massless gauge field B' couples to L_R via $(\tau^3 - \frac{\mathbf{1}}{2})$ which makes sure that B'_μ couples *only* to e_R, being the lower component of L_R. Concentrating on this massless gauge field, the kinetic term of L_R gives schematically:

$$\begin{aligned} \bar{L}_R i\gamma^\mu D_\mu L_R &= \bar{e}_R i\gamma^\mu \partial_\mu e_R - \tilde{g}\bar{e}_R\gamma^\mu e_R B'_\mu + \bar{\nu}_R i\gamma^\mu\partial_\mu \nu_R \\ &+ \text{couplings with heavy gauge bosons,} \end{aligned} \tag{5.20}$$

with $\tilde{g} = \frac{gg'}{\sqrt{g^2+g'^2}}$. This pattern of symmetry breaking is exactly the same as in the SM[6] where the group $SU(2)_L \otimes U(1)_Y$ breaks down to $U(1)$ so that three gauge bosons acquire a mass and one remains massless. This massless field, the photon field, couples to the LH electron, while the neutrino only

[5] This object P has four components each being a 2×2 *matrix*; the order of things does play a role!

[6] The definition of $\tilde{g}$ is very much related to that of e in the SM, see section 2.2.1.

has couplings with the heavier force carriers and does not play a role in QED. In the present case $SU(2)_R \otimes U(1)_{B-L}$ reduces to $U(1)$. If we simply ignore the heavy gauge bosons and the RH neutrino - for the sake of unveiling the structure of the massless theory we are left with, as in QED - we can write (5.20) as

$$\bar{e}_R i\gamma^\mu D_\mu e_R := \bar{e}_R i\gamma^\mu(\partial_\mu + i\tilde{g}B'_\mu)e_R.$$

Now let us have a look at the LH fermion kinetic terms. From equation (5.18) we find $B = -\sin\theta Z_R + \cos\theta B'$ which enables us to write

$$\begin{aligned}\bar{L}_L i\gamma^\mu D_\mu L_L &= \bar{L}_L i\gamma^\mu(\partial_\mu + igA^L_\mu - \tfrac{1}{2}i\tilde{g}B'_\mu)L_L \\ &\quad + \text{couplings with heavy gauge bosons.} \end{aligned} \tag{5.21}$$

The last two equations show that the massless part of the theory is exactly the same as in the standard model prior to its symmetry breaking.

5.3.2 Second Stage: $G_{SM} \to U(1)_{QED}$

At lower energies the two other Higgs fields will also have a nonzero expectation value. To be precise:

$$\begin{aligned}\langle\Phi\rangle &= \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} \\ \langle\Delta_L\rangle &= \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}. \end{aligned} \tag{5.22}$$

At this stage the LH gauge bosons get their masses. Also, some of the LH fermions get a Majorana mass and the Dirac masses are produced. As promised, the fermions will be discussed later. For now, we concentrate on the gauge bosons.

In the previous section we saw that after the first stage the massless part of the theory possessed the all too familiar $SU(2) \otimes U(1)$ structure of the standard model. We will work out what happens to these massless parts of the theory to get a feeling of what happens if v_R is by far the biggest scale in the theory, so that particles with large masses acquired at the first stage essentially decouple from the others.

The mass matrix in the basis $(A^{L,1}, A^{L,2}, A^{L,3}, B')$ is given by:

$$\frac{1}{4}\begin{pmatrix} g^2x & & & \\ & g^2x & & \\ & & g^2y & -g\tilde{g}y \\ & & -g\tilde{g}y & \tilde{g}^2y \end{pmatrix}, \tag{5.23}$$

where $x := \kappa^2 + \kappa'^2 + 2v_L^2$ and $y := x + 2v_L^2$. The eigenvectors are completely analogous to those in previous section and are given in table 5.2.

Eigenvector	Mass	Field combination	New name
$(1, i, 0, 0)$	$g\sqrt{x/2}$	$\frac{1}{\sqrt{2}}(A_\mu^{L,1} + iA_\mu^{L,2})$	$W_{L,\mu}^-$
$(1, -i, 0, 0)$	$g\sqrt{x/2}$	$\frac{1}{\sqrt{2}}(A_\mu^{L,1} - iA_\mu^{L,2})$	$W_{L,\mu}^+$
$(0, 0, g, -\tilde{g})$	$\sqrt{(g^2+\tilde{g}^2)y/2}$	$\frac{1}{\sqrt{g^2+\tilde{g}^2}}(gA_\mu^{L,3} - \tilde{g}B'_\mu)$	$Z_{L,\mu}$
$(0, 0, \tilde{g}, g)$	0	$\frac{1}{\sqrt{g^2+\tilde{g}^2}}(\tilde{g}A_\mu^{L,3} + gB'_\mu)$	A_μ

Table 5.2: The massive combinations of the 'standard model gauge bosons'. The field B' is the massless field from the first redefinition from table 5.1.

To make sure this point is clear: only the gauge fields that remained massless in the previous section, are dealt with in the above table. In this approximation the *massive* combinations from table 5.1 are untouched by the second breaking stage. A more precise study of the massive eigenstates of the mass matrix show that the charged gauge bosons $W_L^\pm$ have a tiny RH admixture, and vice versa. This effect is also ignored in the above which is reasonable as long as $\kappa \gg \kappa'$.

In our notation, in particular the use of B', the similarities between the two stages is stressed, but it is not so clear how the fields from the second table can be written in terms of the original fields. The article of Senjanovic and Mohapatra [20], presents a somewhat different notation of the massive combinations. The equivalent of the Weinberg angle from the SM is introduced:

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+2g'^2}} \qquad \cos\theta_W = \sqrt{\frac{g^2+g'^2}{g^2+2g'^2}} \tag{5.24}$$

Two useful consequences of this are $\sqrt{\cos 2\theta_W} = g/\sqrt{g^2+2g'^2}$ and $\tan\theta_W =$

$g'/\sqrt{g^2+g'^2}$. Now we can rewrite the massive combinations and their masses as follows:

$$
\begin{aligned}
A_\mu &= \sin\theta_W(A_\mu^{L,3}+A_\mu^{R,3})+(\cos 2\theta_W)^{1/2}B_\mu,\\
Z_{L,\mu} &= \cos\theta_W A_\mu^{L,3}-\sin\theta_W\tan\theta_W A_\mu^{R,3}-\tan\theta_W(\cos 2\theta_W)^{1/2}B_\mu,\\
Z_{R,\mu} &= \frac{(\cos 2\theta_W)^{1/2}}{\cos\theta_w}A_\mu^{R,3}-\tan\theta_w B_\mu\\
W_{L,\mu}^{\pm} &= \frac{1}{\sqrt{2}}(A^{L,1}\mp iA^{L,2})\\
W_{R,\mu}^{\pm} &= \frac{1}{\sqrt{2}}(A^{R,1}\mp iA^{R,2})
\end{aligned}
\tag{5.25}
$$

The same can be done for their masses

$$
\begin{aligned}
m_A &= 0,\\
m_{Z_L}^2 &= \frac{g^2}{2}\frac{1}{\cos^2\theta_W}(\kappa^2+\kappa'^2+4v_L^2),\\
m_{Z_R}^2 &= 2(g^2+g'^2)v_R^2,\\
m_{W_L^\pm}^2 &= \frac{g^2}{2}(\kappa^2+\kappa'^2+2v_L^2),\\
m_{W_R^\pm}^2 &= g^2v_R^2.
\end{aligned}
\tag{5.26}
$$

The previous two sets of equations do not give true equalities, for we made the approximations mentioned earlier.

It is of course possible to work out the complete set of mass terms coming from the kinetic terms of the Higgs fields after the two stages of symmetry breaking. In the basis $(A^{L,1},...,A^{R,1},...,B)$ the full matrix is the following:

$$
M=\left(\begin{array}{ccc|ccc|c}
 & & & & & & 0\\
 & M_L & & & M_{LR} & & 0\\
 & & & & & & -gg'v_L^2\\
\hline
 & & & & & & 0\\
 & M_{LR} & & & M_R & & 0\\
 & & & & & & -gg'v_R^2\\
\hline
0 & 0 & -g^2v_L^2 & 0 & 0 & -gg'v_R^2 & g'^2(v_L^2+v_R^2)
\end{array}\right)
\tag{5.27}
$$

with the submatrices

$$\begin{aligned} M_i &= \frac{g^2}{4}\begin{pmatrix} \kappa^2+\kappa'^2+2v_i^2 & 0 & 0 \\ 0 & \kappa^2+\kappa'^2+2v_i^2 & 0 \\ 0 & 0 & \kappa^2+\kappa'^2+4v_i^2 \end{pmatrix} \\ M_{LR} &= -\frac{g^2}{4}\begin{pmatrix} 2\kappa\kappa' & 0 & 0 \\ 0 & 2\kappa\kappa' & 0 \\ 0 & 0 & g^2(\kappa^2+\kappa'^2) \end{pmatrix} \end{aligned} \tag{5.28}$$

The problem here is obvious. It is in principle possible to figure out the eigenvectors and eigenvalues of this matrix. After a short struggle, computer programs will give an answer. It may be no surprise that these are awfully complicated expressions that are hardly possible to handle. Therefore, people mainly work in the approximation adopted in this section.

5.4 The Electric Charge Formula

After the two stages of symmetry breaking their is only one massless gauge field left which means there is still one linear combination of generators that leaves the vacuum invariant. This corresponds to the electric charge.

Let us elaborate a bit on what this means. Suppose we have a matrix scalar field ϕ with $\langle\phi\rangle =: \phi_0$. Now define ϕ' through $\phi = \phi' + \phi_0$. If ϕ transforms like $\phi \to U_L \phi U_R^\dagger$ we may wonder whether we can find a set of transformations that leaves the vacuum invariant:

$$U_L \phi_0 U_R^\dagger = \phi_0. \tag{5.29}$$

Suppose for definiteness that this subset of transformations has one generator, $\tilde{T}$. Then the above condition is equivalent to saying that after symmetry breaking there is one massless gauge field, say $\tilde{A}$, that couples with 'coupling strength' $\tilde{T}$ to the matter fields. This generator $\tilde{T}$ can be identified with the electric charge.

Having made these remarks we are ready to find the electric charge formula. Use the following parametrizations $U_L = \exp[i\alpha_L^a \tau^a]$ and $U_R = \exp[i\alpha_R^a \tau^a]$. For the $U(1)$ transformation we take $\exp[-iq_{B-L}\beta\mathbf{1}/2]$. We need to find a set of (real!) α's and a β such that the equivalent of (5.29) is satisfied for all the Higgs fields in the model. To this end we consider the infinitesimal form of the gauge transformation, starting with $\langle\Delta_R\rangle$:

$$
\begin{aligned}
e^{-i\beta}e^{-i\alpha_R^a\tau^a}\langle\Delta_R\rangle e^{i\alpha_R^a\tau^a} &\approx (1-i\beta)(1-i\alpha_R^a\tau^a)\langle\Delta_R\rangle(1+i\alpha_R^a\tau^a) \\
&\approx \langle\Delta_R\rangle - i[\alpha_R^a\tau^a,\langle\Delta_R\rangle] - i\mathbf{1}\beta\langle\Delta_R\rangle \\
&= \langle\Delta_R\rangle - \frac{iv_R}{2}\begin{pmatrix} \alpha_R^1 - i\alpha_R^2 & 0 \\ 2(\beta-\alpha_R^3) & i\alpha_R^2 - \alpha_R^1 \end{pmatrix}
\end{aligned} \tag{5.30}
$$

With the constraint that the parameters of the transformations must be real, we find $\alpha_R^1 = 0 = \alpha_R^2$ and $\alpha_R^3 = \beta$.

As for the bi-doublet Φ

$$
e^{i\alpha_L^a\tau^a}\langle\phi\rangle e^{-i\alpha_R^a\tau^a} \approx \langle\phi\rangle - i(\alpha_L^a\tau^a\langle\phi\rangle - \langle\phi\rangle\alpha_R^a\tau^a). \tag{5.31}
$$

The second term on the left hand side should vanish, leading to the constraint $\alpha_L^3 = \alpha_R^3$.

All in all we see that transformations with $\alpha_L^3 = \alpha_R^3 = \beta$ are the of type we were looking for. This means the combination $\tau_L^3 + \tau_R^3 + \frac{\mathbf{1}_{B-L}}{2}$ is the only unbroken generator and hence corresponds to the electric charge. People usually write this statement as [20]

$$
Q = T_L^3 + T_R^3 + \frac{B-L}{2}. \tag{5.32}
$$

This is the equivalent of Gell-Mann Nishijima relation for the electric charge in the standard model: $Q = T^3 + Y/2$. We can check the above relation by calculating $Q(\Delta_{\mathcal{H}}) = \tau_L^3\Delta_{\mathcal{H}} - \Delta_{\mathcal{H}}\tau_L^3 - \Delta_{\mathcal{H}}$ and comparing it with the charge assignments in equation (5.13), which now turn out to be consistent.

Note that sofar we did not invoke the vev of the triplet field Δ_L. It seems it is not needed it to break the full gauge group down to $U(1)$ which leaves open the possibility of a vanishing v_L, as may be desirable.

5.5 Lepton Masses

In the previous sections dealt with the Gauge boson masses only. Also of great interest are the lepton masses. We will see that in a fairly natural way a seesaw mechanism can be implemented so that the formulae from section 4.2 are applicable.

To find the mass terms of the leptons that arise in the symmetry breaking process we need to consider the Yukawa terms that couple the lepton fields to the various Higgs fields. At the first stage this is the term $hL_R^T C\epsilon\Delta_R L_R$ and with (5.16) we find that the creation of the Majorana mass term $hv_R(\nu_R^T C\nu_R + \nu_R^\dagger C\nu_R^*)$ is a fact. The second term between brackets is due to the Hermitian conjugate of the Yukawa term mentioned. This "h.c." is always there. In this first stage both the electron and ν_L remain massless.

As for the second stage, now the important Yukawa terms are $f_1\overline{L}_L\Phi L_R + f_2\overline{L}_L\tilde{\Phi}L_R$ and $hL_L^T C\epsilon\Delta_L L_L$. In appendix A on the gamma and sigma matrices the effect of sandwiching any 2×2 matrix can be found explicitly. Equation (5.22) tells us we get the mass terms $hv_L(\nu_L^T C\nu_L + \nu_L^\dagger C\nu_L^*)$ and $(f_1\kappa + f_2\kappa')(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L)$ for the neutrino and $(f_1\kappa' + f_2\kappa)(\bar{e}_L e_R + \bar{e}_R e_L)$ for the electron. By construction of the Higgs potential the electron only has a Dirac mass. This needs to be the case since after both symmetry breaking stages we want the electron to have a U(1) charge: the ordinary electric charge.

To summarize, the following mass terms appears:

$$\begin{aligned}\mathcal{L}_{mass,\nu} &= (f_1\kappa + f_2\kappa')(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L) + hv_L(\nu_L^T C\nu_L - \nu_L^\dagger C\nu_L^*) \\ &+ hv_R(\nu_R^T C\nu_R - \nu_R^\dagger C\nu_R^*). \end{aligned} \tag{5.33}$$

Following section 4.2 we define $\nu := \nu_L$ and $N := (\nu_R)^c$ so that the previous equation can be conveniently written as

$$\mathcal{L}_{mass,\nu} = \frac{1}{2}\begin{pmatrix} \nu^T & N^T \end{pmatrix}\begin{pmatrix} M_L & m \\ m & M_R \end{pmatrix}\begin{pmatrix} C\nu \\ CN \end{pmatrix} + h.c.$$
$$\text{with} \qquad m = f_1\kappa + f_2\kappa', \qquad M_L = 2hv_L, \qquad M_R = 2hv_R. \tag{5.34}$$

From equation (4.4) we find that

$$\begin{aligned} M_\nu &= 2hv_L - \frac{(f_1\kappa + f_2\kappa')^2}{2hv_R} \\ M_N &= 2hv_R \end{aligned} \tag{5.35}$$

From appendix C we know that v_L is roughly of the order κ^2/v_R creating the pleasant situation that if $v_R \to \infty$ the LH neutrino mass goes to zero, while the mass of the extra gauge bosons and the RH neutrino goes to infinity.

5.6 Interactions between Leptons and Gauge Bosons

At the beginning of this chapter we introduced the seven gauge fields. Later we saw what are the physical (i.e. massive) combinations we should work with. In the present section we will give the result of inserting (the inverse of) equation (5.25) in $\bar{L}_{\mathcal{H}} i\gamma^\mu D_\mu L_{\mathcal{H}}$, resulting in numerous interaction terms with leptons and gauge fields. In other words, the interactions will be written in terms of the physical fields.

The charged currents are given by

$$\mathcal{L}^{cc} = -\frac{g}{\sqrt{2}}[\bar{e}_L\gamma^\mu\nu_L W^-_{L,\mu} + \bar{\nu}_L\gamma^\mu e_L W^+_{L,\mu} + \bar{e}_R\gamma^\mu\nu_R W^-_{R,\mu} + \bar{\nu}_R\gamma^\mu e_R W^+_{R,\mu}]. \quad (5.36)$$

The following table (5.3) summarizes the neutral current interactions:

	$\bar{e}_L e_L$	$\bar{\nu}_L \nu_L$
Z_L	$g(\cos\theta_W - \frac{1}{2}\sec\theta_W)$	$-\frac{1}{2}g\sec\theta_W$
Z_R	$-\frac{1}{2}g'\tan\theta_W$	$-\frac{1}{2}g'\tan\theta_W$
A	$g\sin\theta_W$	

Table 5.3: Couplings of the LH leptons to the neutral gauge bosons.

Table 5.4 gives similar results for the RH lepton doublet. The way to read the tables is as follows. Consider the first line of the upper table. It has only one entry, $-\frac{g}{\sqrt{2}}$, which is to say that the corresponding interaction term is $-\frac{g}{\sqrt{2}}\bar{e_L}\gamma^\mu e_L W^-_{L,\mu}$. From the coupling of A_μ with the charged leptons we see that $g\sin\theta_W$ can be identified with the electromagnetic coupling e, as in the standard model.

	$\bar{e}_R e_R$	$\bar{\nu}_R \nu_R$
Z_R	$\frac{g^2-g'^2}{2g}\tan\theta_W$	$-\frac{1}{2}g'\cot\theta_W$
Z_L	$-g\sin\theta_W\tan\theta_W$	
A	$g\sin\theta_W$	

Table 5.4: Couplings of the RH leptons to the neutral gauge bosons.

CHAPTER 6

The LRSM and reality

In the previous chapter we described in some detail the left-right symmetric model. It is of course important to know whether the parameters in the model can be chosen in a way that the correct values of, for example, the lepton and gauge boson masses are produced by it. If possible, this should happen in a natural way. The term natural can be interpreted in different ways. Firstly, an example of an unnatural situation is one in which some parameters need to be chosen extremely accurately, for example the value of the sixth decimal is of crucial importance, is considered unnatural. In other words, finetuning is most undesirable. Secondly, dimensionless parameters should be roughly of order 1. If it needs to have a small value to fit the model to reality, one should look for approximate symmetries in the theory with the property that if the parameter goes to zero, the symmetry becomes exact. The latter interpretation is sometimes referred to as 'naturalness in the sense of 't Hooft'.

6.1 An Estimate of VEV's

6.1.1 Values

We will now make a very straightforward estimate of v_L, R, and $\kappa_+^2 := \kappa^2+\kappa'^2$. In doing so a value is found for the Dirac and Majorana Yukawa couplings (f and h, respectively). An important role in this argument is played by the seesaw relation which follows directly from the minimization of the Higgs potential (see appendix C):

$$v_L v_R = \gamma \kappa_+^2 \quad \text{with} \quad \gamma := \frac{\beta_1 \kappa\kappa' + \beta_2 \kappa^2 + \beta_3 \kappa'^2}{(2\rho_1 - \rho_3)\kappa_+^2} \tag{6.1}$$

Below, γ will be a free parameter on which some of the quantities of interest turn out to depend. Besides (6.1), we will use the following relations (to be found in section 5.5):

$$\begin{aligned} m_\nu &= 2hv_L - \frac{(f_1\kappa + f_2\kappa')^2}{2hv_R} \\ m_e &= f_1\kappa' + f_2\kappa \end{aligned} \tag{6.2}$$

The made assumptions are: (i) $\kappa'/\kappa \ll 1$; $(ii) f := f_1 \approx f_2$; (iii) the two terms in m_ν are both more or less equal to m_ν.

From the W boson mass (≈ 80 GeV) and its formula in the LRSM (5.26) we find that $\kappa \approx \kappa_+ \approx 200$ GeV. Another numerical input is $m_e = 0.5$ MeV and lastly, we estimate $m_\nu \approx 1$ eV, so that in the following calculation both terms occurring in m_ν will be taken of the order of 1 eV.

With assumptions (i) and (ii) we see that $f_1\kappa + f_2\kappa' \approx f_1\kappa' + f_2\kappa \approx f\kappa$ and from the electron mass formula it follows that $f = m_e/\kappa_+ \approx 3 \times 10^{-6}$. Setting the two terms in m_ν to ~ 1 eV (assumption (iii)) gives the final ingredient to express all the unknowns in terms of κ_+, m_e, m_ν and γ. Table 6.1 shows the values of v_R, v_L and h for some values of γ:

$$\begin{aligned} v_L &= \frac{m_\nu \kappa_+}{m_e}\sqrt{\gamma} \approx 4\sqrt{\gamma} \times 10^{-4} \text{ GeV} \\ v_R &= \frac{m_e \kappa_+}{m_\nu}\sqrt{\gamma} \approx \sqrt{\gamma} \times 10^{8} \text{ GeV} \\ f &= 2h\sqrt{\gamma} = \frac{m_e}{\kappa} \approx 3 \times 10^{-6} \\ m_N &= 2hv_R = \frac{m_e^2}{m_\nu} \approx 3 \times 10^{2} \text{ GeV} \end{aligned} \tag{6.3}$$

6.1.2 Discussion

From the previous chapter we know that v_R sets the mass scale of the RH gauge bosons. Direct experimental searches for these heavy extra gauge bosons has resulted in a lower bound of[1] $M_{W_R} > 720$ GeV, [21]. A second

[1]This result holds under the conditions that $m_N \ll M_{W_R}$ and that the model has a (pseudo) manifest LR symmetry. The first is obviously met with in this numerical analysis, and so is the second as follows from as follows from the reality of κ and κ' [22].

γ	v_R (GeV)	v_L (eV)	h
1	1×10^8	4×10^5	1×10^{-6}
10^{-1}	3×10^7	1×10^5	4×10^{-6}
10^{-2}	1×10^7	4×10^4	1×10^{-5}
10^{-4}	1×10^6	4×10^3	1×10^{-4}
10^{-6}	1×10^5	4×10^2	1×10^{-3}
10^{-8}	1×10^4	4×10^1	1×10^{-2}

Table 6.1: The values of some key parameters are dependent on γ.

lower bound was obtained by considering the $K_L - K_S$ mass splitting, resulting in $M_{W_R} > 1.6$ TeV. With v_R as in (6.3) the mass of W_R is well above these lower bounds. As for the other parameters: v_L is small enough not to disturb the value of $\rho_{ew} := M_{W_L}^2/(M_Z \cos\theta_W)^2 = (\kappa_+^2 + 2v_L^2)/(\kappa_+^2 + 4v_L^2)$. It should be within 1% of unity, implying $v_L < 14$ GeV [22].

Looking at table 6.1 shows that all values of γ between 10^{-8} and 1, and perhaps an even wider range, give acceptable results regarding the bounds given above. However, as said earlier, extremely small values (i.e. $\ll 1$) for dimensionless parameters are considered unnatural. In this respect, there appears to be a conflict between γ and h. A value of γ close to 1, requires h to be 10^{-6}. At the value $\gamma = 10^{-4}$ both parameters are of the same order of magnitude and the extra gauge bosons are around 10^6 GeV, which is still two orders of magnitude above the energies at which LHC will be operating (14 TeV [24]).

Note that the mass of the heavy neutrino turns out to be 3×10^2 GeV and is independent of γ. Now the work done in section 5.6 pays off. From equation (5.36) we can see it is not likely that the heavy neutrino, which is predominantly RH up to a tiny admixture of the LH neutrino, is produced in charged current processes at LHC because of the exchange of a heavy W_R ('new') gauge boson. In discussing the massive eigenstates of the gauge boson mass matrix we ignored the fact that (mostly) right handed charged bosons contain a tiny fraction of the LH $SU(2)$ boson. However, the mixing angle is of the order $(M_{W_L}/M_{W_R})^2$ [20] so that also the exchange of W_L is suppressed. The RH neutrino also does not couple to[2] Z_L implying a suppression of the pair production of two RH neutrinos. All this may seem somewhat pessimistic, but in principle it will be possible to produce the RH neutrino so therefore one may expect it to happen.

[2]Here the same holds: the massive neutral mass eigenstates are mostly left (right) with a tiny admixture of right (left).

In section 4.1 we touched the subject of neutrinoless double beta decay. We saw the 'standard diagram' which relates the amplitude of the process to the Majorana mass of the neutrino. It was noted that the decay could also take place with the exchange of some heavy scalar. In the model here under consideration we indeed have a scalar at our disposal capable of producing lepton number violating processes : δ_L. In the appendix to [22] we see that its mass is of the order v_R. Figure 6.1 shows the Feynman diagram of the process.

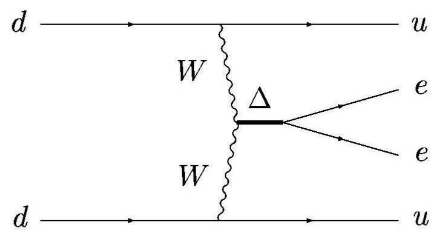

Figure 6.1: Feynman diagram of neutrinoless double beta decay mediated by the heavy scalar Δ_L.

We would like to estimate the ratio of this contribution and the one from figure 4.2. We do not need to bother with the common features but with the differences of the diagrams. Therefore from figure 6.2 we should consider the $WW\Delta$- and Δee-vertices and the Δ-propagator. The first vertex comes from $\mathrm{tr}(D_\mu\Delta)^\dagger(D^\mu\Delta)$ and from equations (5.9) and (5.22) it can be seen that it gives a factor (in magnitude) v_L. The second vertex is produced by the Yukawa coupling, given in equation (5.6), and gives h^{ee}. Finally, the propagator is just $1/m_\Delta^2$, since the energies involved are much lower than m_Δ. We conclude that the magnitude of the relevant part of figure is

$$\mathcal{M}_\Delta = \frac{hv_L}{m_\Delta^2}$$

Turning to the amplitude of figure 4.2, we see that the relevant part consists of twice a $W\nu e$-vertex, a mass insertion m_ν and two light-neutrino propagators with momentum, say, p. The vertex gives a factor $g/\sqrt{2}$ (see table section 5.6) and the propagator $1/p$. All in all, for the relevant part of figure 4.1 we find

$$\mathcal{M}_\nu = \frac{g^2 m_\nu}{2p^2}$$

A short note is in place here. In the literature it is often said that the diagram is proportional to the effective Majorana mass, usually denoted by

$m_{\beta\beta}$ (for example [25]). If one deals with the three generation case correctly, the MNS-matrix starts to play a role. There will be a diagram for each of the three light neutrinos (ν_i) and for both vertices in each diagram there is a factor U_{ei}^{MNS} so that summing up the diagrams (ie. summing over i) we automatically encounter the quantity $m_{\beta\beta} := \sum_i (U_{ei}^{MNS})^2 m_{\nu_i}$. It is this quantity that can be inferred from experiments.

Realizing that in the approximation we made earlier $hv_L \approx m_\nu$, and leaving out g and factors of two, we find the ratio

$$\frac{\mathcal{M}_\Delta}{\mathcal{M}_\nu} \approx \frac{p^2}{m_\Delta^2}.$$

It is reasonable to take p in the range of the energy electrons emitted in the decay process. A value around 1 MeV [23] gives the ratio the extremely small value of 10^{-11}, indicating that contributions from decay via the alternative channel can be completely ignored.

6.2 Alternative approaches

The values found in the previous section imply the masses of the extra gauge bosons to be extremely high; around 10^8 GeV. If this is the case there is no chance to detect them in the near future. Therefore, reasonable effort has been put in trying to lower the value v_R.

One way to achieve this is to take the β parameters in the Higgs small, or zero. In that case equation (6.1) allows the product $v_L v_R$ to be much smaller, or zero. But since the β-terms in the potential are allowed by the gauge and parity symmetry, it would be strange to just suppress them or set them to zero. It would be more natural to impose an extra symmetry on the Lagrangian that causes this to happen.

A simple example is to demand invariance under $\Delta_L \to \Delta_L$ and $\Delta_R \to -\Delta_R$. The good thing is it excludes the β terms ($\beta_i = 0$), the bad thing is that Majorana mass terms are no longer allowed. At first sight only the RH Majorana mass seems to be forbidden but the parity symmetry carries this over to the LH mass term. Some authors try to achieve the goal by a so called horizontal symmetry.[3] The idea is that the β's will be suppressed by powers of some small quantity. The mass scale of the RH gauge bosons can be brought down to detectable values.

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[3]For a treatment of embedding this in a LRSM: [26] or for a more general treatment [27].

CHAPTER 7

Conclusion

We started with the observation that in the standard model the right handed neutrino is completely redundant. The combination of describing the neutrino as a massless particle and the V-A nature of the weak interaction are enough to ensure this.

The introduction of the right handed neutrino is as vital for the possibility of a Dirac mass, as is it is for the implementation of the seesaw mechanism. The seesaw mechanism is the name of the process in which the interplay of a large Majorana mass for the right handed neutrino and a small Dirac mass ensure the appearance of one heavy and one light Majorana neutrino. As this is considered a very realistic scenario for small masses, the introduction of the right handed neutrino seems inevitable.

This can be done within the context of the standard model by simply adding to the Lagrangian the field's kinetic terms, a Majorana mass term and the correct Yukawa coupling (4.12) for producing the Dirac mass. It turns out that the right handed neutrino must be a singlet under the local symmetry group standard model, hence the often used 'sterile neutrino'. The problem of this approach is that there is no hint whatsoever to the order of magnitude of M_R. A second method to produce a mass for the neutrino, a Majorana mass for the left handed field this time, is by adding a triplet scalar field. In this scenario there is no need for a right handed neutrino, excluding a seesaw scenario and with that any explanation of the smallness of the masses.

It was decided to study the a left right symmetric model in more detail. It obviously offers room to the right handed neutrino and by some sense it provides an explanation of the parity violation of the weak interactions. Furthermore, it has the pleasant property that a seesaw mechanism more or less automatically arises, encompassing both of the above mentioned scenarios for obtaining neutrino masses.

For this to work properly it was found that the extra gauge bosons have the somewhat disturbingly high mass of $10^3 - 10^5$ TeV, and all bounds from experiments seem to be fulfilled. The high value for the mass is disturbing in the sense that any chance of detection in the near future is excluded. The most important assumptions that were made are the β and rho parameters in the scalar potential are of order unity so that the relation $v_L v_R = \kappa_+^2$ was to be fulfilled. Much lower values of the extra bosons can be achieved by forcing the β parameters to be small. People have achieved this by adding additional symmetries to the left-right symmetric model.

The numbers found in the numerical exercise were used to find out to what extend one should take into account an alternative channel in the process of neutrinoless double beta decay. A heavy scalar triplet can be produced in the merging of two Z_L bosons and then decay into e^+e_- (see figure 6.1). It was found that the contribution of this channel is 10^{-11} times smaller than the one depicted in figure 4.2, going through the exchange of a Majorana neutrino. It is this process that people use to find the so-called effective mass of the neutrino. Neglecting effects of other channels seems to be well justified within the set of the assumptions made here.

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CHAPTER 8

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APPENDIX A

Pauli and Dirac Matrices

In this appendix the main properties of both the Pauli and Dirac matrices are listed. The latter are often called gamma matrices.

A.1 Pauli matrices

The famous Pauli matrices are given by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

These matrices are Hermtitian and obey the multiplication rule $\sigma^i \sigma^j = \delta^{ij} + i\epsilon^{ijk}\sigma^k$ so that the matrices $\tau^i := \sigma^i/2$ satisfy

$$[\tau^i, \tau^j] = i\epsilon^{ijk}\tau^k \qquad \text{and} \qquad \{\tau^i, \tau^j\} = \frac{\delta^{ij}}{2}.$$

From $\sigma^2 \sigma^i \sigma^2 = -\sigma^{i*}$ it follows that

$$\sigma^2 \sigma^\mu \sigma^2 = \bar{\sigma}^{\mu*} \qquad \text{and} \qquad \sigma^2 \bar{\sigma}^\mu \sigma^2 = \sigma^{\mu*}.$$

In gauge theories one often encounters the following matrix

$$A^i \tau^i = \frac{1}{2} \begin{pmatrix} A^3 & A^1 - iA^2 \\ A^1 + iA^2 & -A^3 \end{pmatrix}$$

Suppose the real numbers $a_0, ..., a_4$ are normalized so that $a_0^2 + ... + a_3^2 = 1$ then the Hermicity and the multiplication rule of the Pauli matrices tells us that

$$a_0 1 + i a_i \sigma^i = \begin{pmatrix} a_0 + i a_3 & a_2 + i a_1 \\ a_2 + i a_1 & a_0 - i a_3 \end{pmatrix} =: U$$

is a unitary matrix. In fact, it is the most general 2×2 unitary matrix. From both expressions for U in the above equations one can see that

$$\sigma^2 U \sigma^2 = U^*.$$

This can be found as well by realizing that

$$\sigma^2 \begin{pmatrix} a & b \\ c & d \end{pmatrix} \sigma^2 = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}.$$

A.2 Dirac matrices

First, define [1]

$$\sigma^\mu = (1, \sigma^i) \qquad \text{and} \qquad \overline{\sigma}^\mu = (1, -\sigma^i)$$

With 1 being the 2×2 unit matrix. In the so-called chiral basis the Dirac (or often called 'gamma') matrices are defined as follows:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}$$

Besides these four gamma matrices, people usually define a fifth. Namely

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

A few rules that are easy to derive:

$$\begin{aligned} \{\gamma^\mu, \gamma^\nu\} &= 2g^{\mu\nu} \\ \{\gamma^5, \gamma^\mu\} &= 0 \\ \gamma^0\gamma^\mu\gamma^0 &= \gamma^{\mu\dagger} \end{aligned}$$

Where we use the metric $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

[1] This notation is borrowed from [5].

APPENDIX B

Conventions for Gauge Transformations and Covariant Derivatives

In this appendix some conventions regarding the signs occurring in the gauge field transformations and covariant derivatives are stated and motivated. They are used mainly in section

The following example covers all the cases that are present in this report. All the fields appearing are possibly matrices, so we must keep an eye on the ordering. Consider a field ϕ that transforms like:

$$\boxed{\phi \to \mathrm{U}\phi\mathrm{V}^\dagger} \tag{B.1}$$

where U and V are elements of some unitary gauge group. We will show that the covariant derivative

$$\boxed{D_\mu\phi := \partial_\mu\phi - igA_\mu^{\mathrm{U}}\phi + ig'\phi A_\mu^{\mathrm{V}}} \tag{B.2}$$

gives rise to acceptable transformation rules of the gauge fields $A_\mu^{\mathrm{U,V}}$ when we demand that this derivative transforms the way ϕ does:

$$\text{if} \qquad (\phi, A_\mu^{\mathrm{U,V}}) \to (\mathrm{U}\phi\mathrm{V}^\dagger, A_\mu'^{\mathrm{U,V}}) \qquad \text{then} \qquad D_\mu\phi \to \mathrm{U}D_\mu\phi\mathrm{V}^\dagger. \tag{B.3}$$

Writing out this condition explicitly[1] (first the 'if'):

[1]Notation used: $\phi' := \mathrm{U}\phi\mathrm{V}$ and $\phi_{,\mu} := \partial_\mu\phi$.

$$\phi'_{,\mu} - igA'^{\mathrm{U}}_{\mu}\phi' + ig'\phi' A'^{\mathrm{V}}_{\mu} = \mathrm{U}_{,\mu}\phi\mathrm{V}^{\dagger} + \mathrm{U}\phi_{,\mu}\mathrm{V}^{\dagger} + \mathrm{U}\phi\mathrm{V}^{\dagger}_{,\mu} - igA'^{\mathrm{U}}_{\mu}\phi' + ig'\phi' A'_{\mu}. \quad \text{(B.4)}$$

Using the 'then' and the RH side of (B.4) this gives the equality

$$\mathrm{U}_{,\mu}\phi\mathrm{V}^{\dagger} + \mathrm{U}\phi_{,\mu}\mathrm{V}^{\dagger} + \mathrm{U}\phi\mathrm{V}^{\dagger}_{,\mu} - igA'^{\mathrm{U}}_{\mu}\phi' + ig'\phi' A'^{\mathrm{V}}_{\mu} = \mathrm{U}(\phi_{,\mu} - igA^{\mathrm{U}}_{\mu}\phi + ig'\phi A^{\mathrm{V}}_{\mu})\mathrm{V}^{\dagger}. \quad \text{(B.5)}$$

The terms with the derivative on ϕ cancel out immediately. Now put in $1 = \mathrm{U}^{\dagger}\mathrm{U} = \mathrm{V}^{\dagger}\mathrm{V}$ in those places in (B.5) so that ϕ occurs only in the combination $\phi' = \mathrm{U}\phi\mathrm{V}^{\dagger}$. Upon regrouping we find

$$(\mathrm{U}_{,\mu}\mathrm{U}^{\dagger} - igA'^{\mathrm{U}}_{\mu} + ig\mathrm{U}A^{\mathrm{U}}_{\mu}\mathrm{U}^{\dagger})\phi' = \phi'(-\mathrm{V}_{,\mu}\mathrm{V}^{\dagger} - igA'^{\mathrm{V}}_{\mu} + ig\mathrm{V}A^{\mathrm{V}}_{\mu}\mathrm{V}^{\dagger}). \quad \text{(B.6)}$$

Since U and V are independent of each other, both sides must be a constant. The easiest option is zero, yielding the transformation rule[2]:

$$\boxed{A'^{\mathrm{U}}_{\mu} = \mathrm{U}A^{\mathrm{U}}_{\mu}\mathrm{U}^{\dagger} - \frac{i}{g}(\partial_{\mu}\mathrm{U})\mathrm{U}^{\dagger}} \quad \text{(B.7)}$$

Using $0 = (\mathrm{V}^{\dagger}\mathrm{V})_{,\mu} = \mathrm{V}^{\dagger}_{,\mu}\mathrm{V} + \mathrm{V}^{\dagger}\mathrm{V}_{,\mu}$ one arrives at a completely analogous expression for A'^{V}_{μ}.

We have seen that the three boxed equations are mutually consistent. It can very well be that other people use different conventions. For example, replacing g by $-g$ gives the results as in [3]. We can apply the above results to fields that transform according to the rule: $\phi \rightarrow \mathrm{U}\phi$ by leaving out the V's and the corresponding gauge fields.

Of special interest are the $U(1)$ transformations. Suppose we have for a field with charge q

$$\phi \rightarrow \mathrm{U}\phi = e^{-iq\alpha(x)}\phi \quad \text{(B.8)}$$

In this case the gauge field is not a matrix so it simply commutes with U and $\mathrm{U}^{\dagger}$. Applying the rule from (B.7) we find

$$A'_{\mu} = A_{\mu} - \frac{q}{g}\partial_{\mu}\alpha. \qquad \text{(wrong!)}$$

This is of course unacceptable. The transformation of the gauge field cannot dependent on the charge of the field ϕ. In QED for example, there is one

[2]Sometimes this is suggestively written as $A'^{\mathrm{U}}_{\mu} = \mathrm{U}\frac{i}{g}(\partial_{\mu} - igA_{\mu})\mathrm{U}^{\dagger}$ (See for example [5]) so that some sort of covariant derivative appears.

gauge field with one and only one transformation rule. The inconsistency can be remedied by redefining the coupling constant g. A factor q should be pulled out: replace g by qg to get this consistent (and acceptable) set of equations:

$$\boxed{\phi \rightarrow e^{-iq\alpha}\phi \quad \Big| \quad D_\mu\phi := (\partial_\mu - iqgA_\mu)\phi \quad \Big| \quad A_\mu \rightarrow A_\mu - \frac{1}{g}\partial_\mu\alpha} \tag{B.9}$$

The same remark as earlier holds: there are different sign conventions possible.

APPENDIX C

The Higgs Potential

In the left-right symmetric model one should write down the most general Higgs potential in the sense that it is invariant under the symmetries of the model. In this case $SU(2)_L \times SU(2)_R \times U(1)$ and in addition the parity symmetry operation as described in equation (5.8). The expression

$$
\begin{aligned}
&V(\phi, \Delta_L, \Delta_R) = \\
&-\mu_1^2[\mathrm{tr}(\phi^\dagger\phi)] - \mu_2^2[\mathrm{tr}(\tilde{\phi}\phi^\dagger) + \mathrm{tr}(\tilde{\phi}^\dagger\phi)] - \mu_3^2[\mathrm{tr}(\Delta_L\Delta_L^\dagger) + \mathrm{tr}(\Delta_R\Delta_R^\dagger)] \\
&+\lambda_1[\mathrm{tr}(\phi\phi^\dagger)]^2 + \lambda_2\{[\mathrm{tr}(\tilde{\phi}\phi^\dagger)]^2 + [\mathrm{tr}(\tilde{\phi}^\dagger\phi)]^2\} + \lambda_3[\mathrm{tr}(\tilde{\phi}\phi^\dagger)\mathrm{tr}(\tilde{\phi}^\dagger\phi)] \\
&+\lambda_4\{\mathrm{tr}(\phi\phi^\dagger)[\mathrm{tr}(\tilde{\phi}\phi^\dagger) + \mathrm{tr}(\tilde{\phi}^\dagger\phi)]\} \\
&+\rho_1\{[\mathrm{tr}(\Delta_L\Delta_L^\dagger)]^2 + [\mathrm{tr}(\Delta_R\Delta_R^\dagger)]^2\} + \rho_2[\mathrm{tr}({\Delta_L}^2)\mathrm{tr}({\Delta_L^\dagger}^2) + \mathrm{tr}({\Delta_R}^2)\mathrm{tr}({\Delta_R^\dagger}^2)] \\
&+\rho_3[\mathrm{tr}(\Delta_L\Delta_L^\dagger)\mathrm{tr}(\Delta_R\Delta_R^\dagger)] + \rho_4[\mathrm{tr}({\Delta_L}^2)\mathrm{tr}({\Delta_R^\dagger}^2) + \mathrm{tr}({\Delta_L^\dagger}^2)\mathrm{tr}({\Delta_R}^2)] \\
&+\alpha_1\{\mathrm{tr}(\phi\phi^\dagger)[\mathrm{tr}(\Delta_L\Delta_L^\dagger) + \mathrm{tr}(\Delta_R\Delta_R^\dagger)]\} + \alpha_2[\mathrm{tr}(\phi\tilde{\phi}^\dagger)\mathrm{tr}(\Delta_R\Delta_R^\dagger) + \mathrm{tr}(\phi^\dagger\tilde{\phi})\mathrm{tr}(\Delta_L\Delta_L^\dagger)] \\
&+\alpha_2^*[\mathrm{tr}(\phi^\dagger\tilde{\phi})\mathrm{tr}(\Delta_R\Delta_R^\dagger) + \mathrm{tr}(\tilde{\phi}^\dagger\phi)\mathrm{tr}(\Delta_L\Delta_L^\dagger)] + \alpha_3[\mathrm{tr}(\phi\phi^\dagger\Delta_L\Delta_L^\dagger) + \mathrm{tr}(\phi\phi^\dagger\Delta_R\Delta_R^\dagger)] \\
&+\beta_1[\mathrm{tr}(\phi\Delta_R\phi^\dagger\Delta_L^\dagger) + \mathrm{tr}(\phi^\dagger\Delta_L\phi\Delta_R^\dagger)] + \beta_2[\mathrm{tr}(\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger) + \mathrm{tr}(\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger)] \\
&+\beta_3[\mathrm{tr}(\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger) + \mathrm{tr}(\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger)]
\end{aligned}
\tag{C.1}
$$

is claimed to be the most general (renormalizable) potential one can think of, consistent with the symmetries [22]. The minimization of this potential leads to some very interesting relations between the different vacuum expectation values. The procedure, as described in [22] is as follows. Insert in the potential the vacuum expectation values

$$\langle\phi\rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} \quad \text{and} \quad \langle\Delta_{L,R}\rangle = \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix}, \tag{C.2}$$

which gives $\tilde{V}(v_L, v_R, \kappa, \kappa') := V(\langle\phi\rangle, \langle\Delta_L\rangle, \langle\Delta_R\rangle)$. Written out in full:

$$\begin{aligned} \tilde{V}(v_L, v_R, \kappa, \kappa') = \quad & -\mu_1^2(\kappa^2+\kappa'^2) - 4\mu_2^2\kappa\kappa' - \mu_3^2(v_L^2+v_R^2) \\ & +\lambda_1(\kappa^2+\kappa'^2)^2 + (8\lambda_2+4\lambda_3)\kappa^2\kappa'^2 + 4\lambda_4\kappa\kappa'(\kappa^2+\kappa'^2) \\ & +\rho_1(v_L^4+v_R^4) + \rho_3 v_L^2 v_R^2 \\ & +[\alpha_1(\kappa^2+\kappa'^2) + 2(\alpha_2+\alpha_2^*)\kappa\kappa' + \alpha_3\kappa'^2](v_L^2+v_R^2) \\ & +2[\beta_1\kappa\kappa' + \beta_2\kappa^2 + \beta_3\kappa'^2]v_L v_R \end{aligned} \tag{C.3}$$

Note that the left-right symmetry is very clear in this expression. We introduce the notation $V_{v_L} = \partial\tilde{V}/\partial v_L$ etc. to denote the first derivatives. The seesaw relation between the v's and κ's mentioned in the main text can be found by simply computing $v_R V_{v_L} - v_L V_{v_R}$. In this particular combination μ_3^2 and all α's are simultaneously eliminated. This can be understood by realizing that the terms in (C.3) containing μ_3^2 and α's, all depend on $v_{L,R}$ through $(v_L^2 + v_R^2)$. Equating this to zero yields:

$$(2\rho_1 - \rho_3)v_L v_R = \beta_1\kappa\kappa' + \beta_2\kappa^2 + \beta_3\kappa'^2 \tag{C.4}$$

In the literature people sometimes define the factor[1]

$$\gamma := \frac{\beta_1\kappa\kappa' + \beta_2\kappa^2 + \beta_3\kappa'^2}{(2\rho_1-\rho_3)\kappa_+^2} \quad \text{with} \quad \kappa_+^2 = \kappa^2 + \kappa'^2,$$

so that the seesaw relation (C.4) can be rewritten as

$$v_L v_R = \gamma\kappa_+^2. \tag{C.5}$$

This equation severely suppresses the freedom of choice for the values of $v_{L,R}$ and κ_+. Since $v_L \ll \kappa_+ \approx 200$ GeV (see section 6.1), the previous equation forces v_r to some high value.

The Higgs potential is a potential source of CP violation. First of all, its parameters (such as α, etc.) can have nonzero imaginary parts. This

[1]If one works in the limit $\kappa \gg \kappa'$, this reduces to $\gamma = \beta_2/(2\rho_1 - \rho_3)$, in accordance with the article of Mohapatra [Phys. Rev **D**23,165 (1981)].

is mostly referred to as explicit CP violation. Secondly, if there is no explicit CP violation and all parameters are real, the vev's could still acquire a complex phase. People call this spontaneous CP violation. By appropriate redefinitions of the fields one can in general choose all vev's but v_L and k_2 real. These two should be replaced by $v_L e^{i\theta_L}$ and $k_2 e^{i\theta_2}$. This leads to two extra minimalization conditions, leading to the appearance of some sines and cosines relations such as (C.4). In [22] it is argued that spontaneous CP violation is excluded in the absence of explicit CP violation, meaning that if one chooses real parameters in the Higgs Potential the vacuum expectation values can generally be taken real.

APPENDIX D

Majorana Equation

It is common to work with the Dirac Lagrangian and its corresponding equation of motion. But what kind of equation of motion, one with two components, do we have for Majorana particles? A good point to start to answer this question is the Dirac equation

$$(i\gamma \cdot \partial - m)\psi = 0 \tag{D.1}$$

Now insert a Majorana spinor into the equation (a Dirac spinor with two identical Weyl spinors, say a). Using the explicit form of the γ's (see appendix A) we see two equations (two components each) emerge. They are, however, connected through complex conjugation and right multiplication with $\epsilon = i\sigma^2$. The lower two components of (D.1) read:

$$i\overline{\sigma} \cdot \partial a - m\epsilon a^* = 0. \tag{D.2}$$

This equation is known as the Majorana equation.

Now we get to the Lagrangian that produces this equation of motion. Just take the Dirac Lagrangian and write it explicitly in terms of its Weyl spinors

$$\mathcal{L} = a^\dagger \overline{\sigma}^\mu \partial_\mu a - b^T \epsilon \sigma^\mu \epsilon \partial_\mu b^* + m(b^T \epsilon a - a^\dagger \epsilon b^*) \tag{D.3}$$

In the case of a Majorana spinor ($a = b$) this reduces to the Lagrangian that produces (D.1). To simplify it somewhat, one can transpose the second term in (D.3), leaving everything unchanged since it is a number, to find that it is identical to the first term. This is not very obvious: one needs to get rid of the two ϵ's in the second term. It can be accomplished by using $\epsilon\sigma^i\epsilon = \sigma^{i*}$. This, together with $\epsilon^2 = -1$, implies $\epsilon\sigma^\mu\epsilon = (-1, \sigma^{i*}) = -\overline{\sigma}^{\mu*}$. Transposing removes the star because of the hermicity of the Pauli matrices. All in all we find:

$$\mathcal{L} = a^\dagger\overline{\sigma}^\mu\partial_\mu a + \tfrac{1}{2}m(a^T\epsilon a - a^\dagger\epsilon a^*) \tag{D.4}$$

where the overall factor of $\frac{1}{2}$ is conventional.