The Cosmic Microwave Background radiation: determining important cosmological parameters

Rianne Jansen 1381040

June 2006

Abstract

The Cosmic Microwave Background radiation (CMB) has been discovered by Penzias and Wilson in 1965. In the first part of this bachelor thesis some basic properties of the CMB will be discussed, as the origin, the spectrum, the temperature and the amount of radiation. In the second part the temperature fluctuations of the CMB will be discussed. The origin of these fluctuations will be eplained and also the way they present information about cosmological parameters. At last there will be given a short overview of the experiments that are done to observe the CMB and what the values of the cosmological parameters are obtained from these data.

Contents

1 Basic properties of the CMB

1.1 Introduction

Nowadays the Big Bang model is the most commonly accepted theory about the origin of the universe. In this bachelor thesis one important aspect of the Big Bang model, the Cosmological Microwave Background radiation (CMB), will be discussed. There will be given a historical overview of the development of the Big Bang model and the discovery of the CMB. In the first part of this thesis some basis properties of the CMB will be explored; the origin of the CMB, its black body spectrum, the temperature of the CMB and the horizon problem. Also the amount of energy of the CMB will be compared with the total amount of radiation energy with falls on earth. The second part of this thesis will deal with the more advanced topic of the anisotropies of the CMB. There will also be discussed how one can determine cosmological parameters from these anisotropies.

1.2 Historical overview

Looking at the origin of the Big Bang model one first comes at the general relativity of Albert Einstein in 1915. From this theory the Friedmann equation can be derived and this equation implies an non-static universe. However Einstein himself was convinced of a static universe and therefore he introduced the so called cosmological constant in another version of his theory of general relativity. This cosmological constant was only a mathematical trick to get the wanted result, a static universe. In 1922 the Russian mathematician Alexander Friedmann did exactly the opposite of what Einstein did. He looked at the more elegant, original theory of general relativity without cosmological constant. He concluded that the universe is expanding. It was the Belgian priest and cosmologist Georges Lemaitre who concluded in 1927 that if the universe is expanding, this must imply that everything started in a so called primeval atom. The Big Bang theory didn't become immediately accepted as common theory about the origin of the universe. There were no supporting proofs from observations for the Big Bang theory and so the most scientists still believed in a static universe. In 1929 Hubble would change this believe with his observation that galaxies are moving away from the earth with a velocity proportional to the distance from the earth; the Hubble's law. This observation convinced Einstein and he even declared this in public. Most scientists still didn't believe in the Big Bang model, one of them was Fred Hoyle. He came in 1949 together with Thomas Gold and Hermann Bondi with the idea of creation of new matter that would fill the holes between the galaxies which are moving away from each other, so that the matter density in the universe remained the same. This theory is called the Steady-State theory. As a kind of a joke Hoyle used the name Big Bang during a radio-program of the BCC in 1950 and that's where the Big Bang

Figure 1: *Penzias and Wilson with their historic horned antenna at Crawford Hill* [20]

model has got its name from. A crucial moment in the battle between the Steady-State theory and the Big Bang model was the coincident observation of the Cosmic Microwave Background radiation (CMB) by Arno Penzias and Robert Wilson. The existence of the CMB was already predicted in 1948 by Ralph Alper and Robert Herman. There are several reasons why in that time no one was going to look for this background radiation. One important reason is that there were only a few scientists with the required knowledge of astronomy, nuclear physics and microwave detection at the same time. Another reason was that the microwave technology was not so far developed yet. Also an reason was the lack of acceptation of the Big Bang model by the greatest part of the astronomers. Penzias and Wilson were looking for something that caused noise in the signal received by their radio telescope and then they discovered, by coincident, the background radiation in 1965. Also Robert Dicke and James Peebles predicted, just before the discovery, the existence of the background radiation, independently of Alpher and Herman. When Penzias and Wilson heard about this prediction, they realised the importance of their discovery [1].

After the discovery of the CMB several experiments has been done to figure out something more about this radiation. Two of the most important experiments has been done with the satellites the Cosmic Background Explorer (COBE) in 1989 and the Wilkinson Microwave Anisotropy Probe (WMAP) in 2001 [1, 2]. In 2007 the Planck satellite will be launched, a successor of COBE and WMAP.

1.3 Basic equations

The evolution of the universe can be described by the following equations: the Friedmann equation and the fluid equation.

The Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8G\pi\rho}{3} - \frac{k}{a^2} \tag{1}$$

The fluid equation:

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0 \tag{2}$$

In these equations the following symbols are used.
p-pressure of the material
ρ-matter density in the universe
G-gravitation constant
$a = a(t)$-scalefactor of the universe
The last symbol, the scalefactor, need to be explained. The following holds: $\vec{r} = a(t)\vec{x}$, where $\vec{r}$ is the distance between two particles (using the term particles for clusters of galaxies) in the universe and $\vec{x}$ the comoving distance, in other words the distance that's carried along with the expansion. The comoving distance hence remains constant during the expansion of the universe. The meaning of the scalefactor is the following; if between two times t_1 and t_2 the value of the scalefactor has doubled in value, then this means the universe has expanded with a factor two.

The density which appears in the Friedmann and in the fluid equation is the total density and exists of the sum of the matter and radiation density. From the fluid equation some relations for this densities can be derived. First it is important to make clear the difference between matter and radiation. With matter we mean non-relativistic particles and with radiation we mean relativistic particles. There will be shown that matter has a negligible pressure, but radiation does exert a pressure. Before relations for the matter and radiation density can be derived, first an expression for this radiation pressure is needed [3].

The radiation pressure

First a general expression for the pressure of a gas in a box will be derived and later one will distinguish between non-relativistic particles (matter) and relativistic particles (radiation). Consider a squared box with sides with length L. The box contains an ideal, isotropic gas which consists of particles with mass m, velocity v en momentum mv. The first thing one need to know is how often the particles with a velocity v_x collide against a wall normal to the x-direction. A particle must travel a distance L and therefore the number of collisions per time will be given by: $\frac{v_x}{2L}$, dividing by two is because the particles can move either in the positive or the negative x-direction and one assumes an isotropic gas. For the change in momentum one has $2mv_x(\frac{v_x}{2l}) = F$. Realize that the direction of the momentum has changed after a collision, therefore the change of momentum contains a factor two. Now one can calculate the pressure for one particle. This can be done by dividing the force through the area of the wall, this will give for the pressure for one particle: $\frac{mv_x^2}{V}$ For the total particles per volume the following holds: $n = \frac{N}{V}$, where N is the total number of particles. Then one get for the total pressure for N particles: $P = mn < v_x^2 >$, where $< v_x^2 >$ is the average velocity of the particles. Because one assumes an isotropic gas the equality $v_x^2 = v_y^2 = v_z^2$ must hold and therefore the pressure can be written in terms of the average velocity $< v^2 >$:

$$P = \frac{1}{3}mn < v^2 >= \frac{1}{3}n < pv > \tag{3}$$

the small p in this equation is the momentum of the particles given by $p = mv$.

Let's first consider the case of non-relativistic particles, for which one assumed approximately holds $P_{mat} = 0$. For the kinetic energy density ϵ the usual equation holds

$$\epsilon = \frac{1}{2}mn < v^2 >$$

together with equation (3) this will give

$$P_{non-rel} = \frac{2}{3}\epsilon = \frac{2}{3}\rho c^2 \frac{v^2}{c^2} \tag{4}$$

For non-relativistic matter $v << c$ and therefore the pressure will indeed be negligible.

Now consider the case of relativistic particles with a velocity $v \approx c$, using formula (3) this will give for the radiation pressure:

$$P_{rad} = \frac{1}{3}n < pc > \tag{5}$$

For the energy density holds:

$$\epsilon_{rad} = \rho_{rad}c^2 = mnc^2 = n < pc > \tag{6}$$

Combining formula (5) and (6) gives the formula for the radiationpressure [4]:

$$P_{rad} = \frac{\rho c^2}{3} \tag{7}$$

Matter and radiation density

Now the formula for the radiation pressure is derived, formula for the matter and radiation density can be obtained from the fluid equation, formula (2). Use

$$P = w\rho c^2, \tag{8}$$

where $w = 0$ corresponds with matter and $w = \frac{1}{3}$ with radiation. Substitution of formula (8) into formula (2) gives:

$$\dot{\rho} + 3\frac{\dot{a}}{a}\rho(1+w) = 0$$

The idea is to rewrite this equation in a clever way with use of the chain rule:

$$\frac{1}{a^3(1+w)}\frac{d}{dt}\left(\rho a^{3(1+w)}\right) = 0$$

This gives:

$$\rho a^3(1+w) = constant$$

and this gives the desired formula for the relation between the density and the scalefactor:

$$\rho \propto \frac{1}{a^4} \tag{9}$$

Fill in $w = \frac{1}{3}$, this gives for the radiation density:

$$\rho_{rad} \propto \frac{1}{a^4} \tag{10}$$

Fill in $w = 0$, this gives for the matter density [3]:

$$\rho_{mat} \propto \frac{1}{a^4} \tag{11}$$

1.4 The origin of the cosmic background radiation

In the beginning the universe was very hot and existed of a plasma of nucleons, free electrons and photons. The photons collided with the free electrons through Thomas scattering and therefore they were not free to travel through the universe and so the universe was totally opaque. During the expansion of the universe, the universe cooled down and the photon energies became less and less. There came a moment for which the photons no longer had enough energy to ionize the nucleons. The moment for which the nucleons and electrons together formed atoms is called recombination. A process which almost instantly follows this recombination is the so called decoupling, the moment for which the photons do not collide any more and therefore are free to travel through the universe. We can observe this radiation, it is called the Cosmic Microwave Background radiation (CMB). The CMB reaches the earth from all directions. This corresponds with the cosmological principle that the universe looks the same everywhere, no matter what the position of the observer is. The cosmological principle is an approximation which becomes more accurate when one looks at larger scales. The CMB has the spectrum of a black-body with a temperature $T_0 = 2.725 \pm 0.0001K$ [3]. A blackbody is a perfect absorber, which absorbs all radiation incident upon it, no matter what the wavelength is [5]. In this case the absorber will be the nucleons and electrons together and instead of absorbing radiation, the photons are already there. But that makes no difference for the spectrum. At the moment of decoupling it is just as if the walls of a black body are suddenly removed. The distribution for the energy density ϵ in a frequency interval f about df will be given by the following formula:

$$\epsilon(f)df = \frac{8\epsilon h}{c^3}\frac{f^3 df}{exp(\frac{hf}{k_b T} - 1)} \tag{12}$$

By integrating formula (12) over all frequencies a formula for the radiation energy density can be obtained:

$$\epsilon_{rad} \equiv \rho_{rad}c^2 = \alpha T^4 \tag{13}$$

where

$$\alpha = \frac{8\pi^5 k_b^4}{15h^3c^3} = 7.5656 \times 10^{-16} Jm^{-3}K^{-4}$$

The fact that the universe cools down during the expansion of the universe has been already mentioned and with our present knowledge it can be derived immediately by combining formula (11) and (13). From these equations one can see:

$$T \propto \frac{1}{a} \tag{14}$$

For the following calculations one assumes a flat universe without cosmological constant. At the moment of decoupling the CMB had a black body spectrum, but what happened with this spectrum in the time from decoupling till now? Did it remain a black body spectrum or did it became completely different? There will be derived that the spectrum indeed remained a black body spectrum, which is in fact not that obvious. One will see this is related to the way in which quantities scale with the scalefactor. If one of these relations would be different, the spectrum would not have remained a black body spectrum during the evolution. The frequency, the temperature and the energy density will change during the evolution of the universe. To see what the effect of the change of these quantities is, it is convenient to express them all in the scalefactor. For the temperature one already has such a formula, formula (14). Formula (13) and (14) combined give an expression for the relation between the energy density and the scalefactor:

$$\epsilon_{rad} \propto T^4 \propto \frac{1}{a^4} \tag{15}$$

From this a relation between the energy itself and the scalefactor can be determined - note the scalefactor is proportional to inverse of the length:

$$\frac{1}{a^4} \propto \epsilon_{rad} = \frac{E}{V} \propto Ea^3$$

and therefore one has:

$$E \propto \frac{1}{a}$$

The frequency is proportional to the energy according to $E = hf$, so now also the frequency can be expressed in terms of the scalefactor:

$$f \propto \frac{1}{a} \tag{16}$$

Look at formula (12). The exponent in the denominator contains the relation f/T. In formula (14) and (16) can be seen that both T and f are proportional to the inverse of the scalefactor, so the denominator remains equal during the expansion of the universe. What about the numerator? From formula (16) one knows $f^3 \propto \frac{1}{a^3}$ and also the left hand side of the equation is proportional to $\frac{1}{a^3}$ so this cancels out against each other. Don't get confused between ϵ_{rad} and $\epsilon(f)$, they differ a factor a .

The conclusion is that the spectrum of the CMB remains a black body spectrum during the expansion of the universe and that is the form of the spectrum as measured today. The temperature of the black body will be lower and the frequency will be redshifted, compared to the spectrum of the photons at decoupling, because the CMB has traveled al the way to earth [3].

1.5 The temperature of the CMB at the decoupling

To estimate the temperature of the CMB at the decoupling one assumes there is one photon needed to ionize one baryon. So the first thing one needs to know is the rate between photons and baryons. Dividing the energy density of equation (13) by the typical energy of a photon $E_{mean} \simeq 3k_BT$ gives for a temperature $T_0 = 2.725K$ the number of photons per volume:

$$n_\gamma = 3.7 \times 10^8 m^{-3} \tag{17}$$

For the number of baryons per unit volume one must know the energy density

$$\epsilon_B = \rho_B c^2 = \Omega_B \rho_c c^2 \simeq 3.38 \times 10^-11 m^{-3} \tag{18}$$

where one uses the experimental data $\Omega_B \simeq 0.02h^{-2}$ where h is the uncertainty in the Hubble constant. The energy density must be divided by the rest mass of protons and neutrons, these rest masses are about $939MeV$. This gives for the number of baryons per unit volume:

$$n_B = 0.22m^{-3} \tag{19}$$

The fraction of photons with energy $\geq I$, where I is the ionization energy of hydrogen equals the Boltzmann factor $exp\left(\frac{-I}{k_BT}\right)$. At the moment of decoupling this fraction approximately equals $\frac{1}{7\times10^9}$, because then there are just enough photons to ionize the baryons, so this is the critical moment. With other words one has the equality [3]

$$exp\left(\frac{-I}{k_BT}\right) = \frac{1}{7\times10^9}$$

This equation can be rewritten to find the temperature at the moment of decoupling

$$T_{dec} = \frac{13.6eV}{k_B ln\,(1.7\times10^9)} \simeq 7400K \tag{20}$$

For the actual temperature at the moment of decoupling one has $T_{dec} \approx 3000K$. A way to obtain this is by using the Saha equation, which will be derived below. Before the decoupling there was equilibrium between the protons, electrons, hydrogen atoms and photons given by the following reaction: $e^- + p \longleftrightarrow H + \gamma$ The ratio for the probability for a bound, P_{bound}, and an unbound state, $P_{unbound}$, is given by the reaction rate

$$\frac{N_eN_p}{N_HN_\gamma} = \frac{P_{bound}}{P_{unbound}} \tag{21}$$

where N_e, N_p, N_H and N_γ are the number of electrons, protons, hydrogen atoms and photons per volume. For the probability of a bound state one has

$$P_{bound} = 2\sum n^2 exp\left(\frac{-E_n}{k_BT}\right) \tag{22}$$

In this equation the 2 is the number of spin states, n^2 is the number of states for a hydrogen atom, $exp(\frac{-E_n}{k_BT})$ is the propability that a state is occupied and the sum is over all n states. Now one rewrites

$$-E_n = -E_1 - (E_n - E_1) = I - (E_n - E_1)$$

with I as the ionization energy. This gives

$$P_{bound} = 2\sum n^2 exp\left(-\frac{(E_n - E_1)}{k_BT}\right) exp\left(\frac{I}{k_BT}\right) \tag{23}$$

where the sum is over all n states. Using the approximation $\frac{E_n-E_1}{k_BT} \gg 1$ for $n > 1$ the sommation term becomes approximately one, so this gives

$$P_{bound} = 2exp\left(\frac{I}{k_BT}\right) \tag{24}$$

For the probability for an electron to be in an unbound state with kinetic energy between E and $E + dE$ the following holds

$$P_{unbound} = 2\frac{4\pi p^2 dp}{h^3} exp\left(\frac{-E}{k_BT}\right) \tag{25}$$

where $E = \frac{p^2}{2m_e}$. The number 2 is the number of spin substates, the factor $\frac{4\pi p^2 dp}{h^3}$ is the number of quantum states per unit volume in the interval between p and $p + dp$ and $exp\left(\frac{-E}{k_B T}\right)$ is the probability for an state to be occupied. Normally for electrons this probability is given by the Fermi-Dirac distribution, but one uses the approximation $E \gg k_B T$ so this reduces to the Maxwell-Boltzmann distribution. Integration of equation (25) over all values of E gives

$$P_{unbound} = 2\sqrt[3]{\frac{2\pi m k_B T}{h^2}} \tag{26}$$

Combining equation (21), (24) and (26) gives

$$\frac{N_p}{N_H} = \frac{N_\gamma}{N_e}\sqrt[3]{\frac{2\pi m k_B T}{h^2}} exp\left(\frac{-I}{k_B T}\right) \tag{27}$$

Now use the following substitutions:
$N_B = N_p + N_H$
$x \equiv$ fraction of ionized hydrogen atoms
$N_e = N_p = x N_B$
$N_H = (1 - x) N_B$ will give the Saha equation

$$\frac{x^2}{1-x} = \frac{1}{N_B}\sqrt[3]{\frac{2\pi m k_B T}{h^2}} exp\left(\frac{-I}{k_B T}\right) \tag{28}$$

Decoupling happens when the fraction of ionized hydrogen atoms x is still very small, from the Saha equation can be calculated this is the case for $3000K < T < 4000K$ [4].

1.6 The temperature of the CMB today

To estimate the temperature of the CMB as one measures on earth today, one must use the Friedmann equation (1) and one must use the fact that the universe was radiation-dominated at early times. This will give from the Friedmann equation (1):

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi \rho_{rad} G}{3} \tag{29}$$

From equation (10) one get:

$$\frac{\dot{\rho}_{rad}}{\rho_{rad}} = -\frac{4\dot{a}}{a} \tag{30}$$

Filling (29) in into (30) gives:

$$\frac{\dot{\rho}_{rad}}{\rho_{rad}} = -4\sqrt{\frac{8\pi \rho_{rad} G}{3}} \tag{31}$$

Write $\dot{\rho}_{rad} = d\rho_{rad}/dt$ and integrate (31)

$$\int \frac{d\rho_{rad}}{\sqrt[3]{\rho_{rad}}} = -4\sqrt{\frac{8\pi G}{3}} \int dt \tag{32}$$

this gives

$$\frac{-2}{\sqrt{\rho_{rad}}} = -4\sqrt{\frac{4\pi G}{3}} t \tag{33}$$

This can be rewritten as

$$c^2 \rho_{rad} = \frac{3c^2}{32\pi G t^2} \tag{34}$$

One also know from Stefan-Boltzmann's law

$$\epsilon = c^2 \rho_{rad} = \alpha T^4 = \frac{\pi^2 (k_B T)^4}{15\hbar^3 c^3} \tag{35}$$

Setting equation (34) equal to equation (35)gives

$$\frac{3c^2}{32\pi G t^2} = \frac{\pi^2 (k_B T)^4}{15\hbar^3 c^3} \tag{36}$$

Rewriting this gives

$$(k_B T)^4 = \frac{45 c^5 \hbar^3}{32\pi^3 G t^2} \tag{37}$$

so when one uses that the age of the universe $t_0 \sim 14Gyr \sim 10^{18}s$, this will give $kT \sim 1meV$, so the temperature will be a few Kelvin. This will be an overestimate since the radiation has cooled more quickly with $t^{-\frac{2}{3}}$ during the matter-dominated era of the universe [4].

From formula (14) one knows the relation between the temperature and the scalefactor. By using this formula one can calculate what the size of the universe was at the moment of decoupling compared to its present size, because both the temperature of the CMB at the time of decoupling, $T_{dec} \approx 3000K$ as the temperature of the CMB now, $T_0 \approx 3K$, are known.

$$\frac{a_{dec}}{a_0} = \frac{T_0}{T_{dec}} \tag{38}$$

Normalize $a_0 = 1$ and filling the known temperatures into equation (38) gives for the scalefactor at the time of decoupling:

$$a_0 \simeq \frac{1}{1000}$$

So the size of our universe today is only a factor thousand larger than the size of the universe at the time of decoupling, which is compared with other numbers in astrophysics quite small [3]!

1.7 Amount of energy of the CMB compared to the total amount of radiation energy

To compare the amount of energy on earth that comes from the CMB with the total radiation energy on earth, one first needs an expression for the total amount of radiation energy on earth. To obtain such an expression one uses two approximations. First one only considers the radiation that comes from the sun and one neglect all other radiation. Second one considers the sun as a black body. The amount of energy which comes from the sun and falls on earth is given by

$$\epsilon_{abs} = \frac{\sigma T_s^4 4\pi R_s^2}{4\pi R_{se}^2} \times 2\pi R_e^2 \tag{39}$$

T_s is the temperature of the sun, R_s is the radius of the sun, R_e is the radius of the earth and R_{se} is the distance between the sun and the earth. The factor σT_s^4 is the Stefan-Boltzmanns law, the factor $\frac{4\pi R_s^2}{4\pi R_{se}^2}$ is radiation emitted by the sun per surface and $2\pi R_e^2$ is the surface of the earth where the radiation is absorbed. Note that only about half of the total of the surface will receive radiation from the sun, which is why one uses 2π instead of 4π. The energy of the sun absorbed by the earth should be equal to the energy emitted by the earth. This energy is given by

$$\epsilon_{em} = \frac{\sigma T_e^4}{4\pi R_e^4} \tag{40}$$

where T_e is the temperature of the earth and R_e is the radius of the earth.

Putting equation (39) equal to equation (40) and rewriting gives

$$T_s^4 = 2\left(\frac{R_s}{R_s e}\right) \times T_e^4 \tag{41}$$

Filling equation (41) in into equation(39) gives a formula for the energy of the sun absorbed by in earth as a function of the temperature of the earth.

$$\epsilon_{abs} = 2\sigma T_e^4 \tag{42}$$

For the energy of the CMB one has

$$\epsilon_{CMB} = \sigma T_{CMB}^4 \tag{43}$$

Combining equations (42) and (43) gives the wanted relationship for the relation between the total amount of energy on the earth with respect to the energy from the CMB:

$$\frac{\epsilon_{abs}}{\epsilon_{CMB}} \propto \left(\frac{T_e}{T_{CMB}}\right)^4 \tag{44}$$

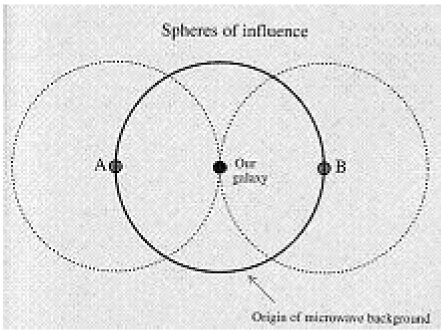

Figure 2: *The horizon problem* [3]

One can also compare the number of photons from the sun and the CMB by using $E_{mean} \simeq 3K_BT$ we can rewrite equation (44) as

$$\frac{n_\gamma^{sum}}{n_\gamma^{CMB}} \propto \left(\frac{T_e}{T_{CMB}}\right)^3 \tag{45}$$

Filling in $T_{CMB} \sim 3K$ and $T_e \sim 3000K$ in equation (44) and (45) gives

$$\frac{\epsilon_{abs}}{\epsilon_{CMB}} \sim 10^{12} \tag{46}$$

and

$$\frac{n_\gamma^{sum}}{n_\gamma^{CMB}} \sim 10^9 \tag{47}$$

So the amount of radiation from the CMB is small compared to the total radiation which falls on the earth.

1.8 The horizon problem

The CMB has the same temperature in every direction - despite little fluctuations. Therefore all the areas of the universe, where the CMB comes from, must once have been in thermal equilibrium. The light that reaches the earth today has traveled a finite distance. This distance is exactly the maximum distance the light may have traveled from the time of decoupling till now. Light which comes from parts of the universe farther away has not been able to reach the earth yet. The surface of those parts of the universe from which the light is just able to reach us is called the surface of last scattering. The CMB one observes today has been traveling since the moment of decoupling. This is the maximum distance the light has been able to travel and therefore it is impossible for the light that reaches the earth from opposite sides from the universe to have been in contact. This becomes clear when we look at figure (2).

The CMB from points A and B can not have been in contact, because the radiation from both point A as point B was 'just' able to reach our galaxy in the time passed from decoupling. [3]

So far one has only looked after two points on opposite sides of the universe, but the situation is even worse. In fact there can be shown that the maximum angle between two points that might have been in thermal equilibrium is about two degrees! At the moment of decoupling the ultimate maximum distance for which two photons could have reached each other is two times the velocity of light. After a time t_0 this fraction of the horizon of $2ct_{dec}$ has expanded to $2ct_{dec}(1 + z_{dec})$ where the factor $1 + z_{dec}$ is due to the expansion of the universe. The redshiftfactor z is defined as

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} \tag{48}$$

where λ_{em} the wavelength of the emitted light is and λ_{obs} the wavelength of the observed light. One can rewrite equation (17) the following way:

$$1 + z = \frac{\lambda_{obs}}{\lambda_{em}} \tag{49}$$

Now combine formula (16) with $\lambda = \frac{c}{f}$ to see

$$\lambda \propto a \tag{50}$$

From equation (18) together with (19) one gets

$$1 + z = \frac{a_{obs}}{a_{em}} \tag{51}$$

So the factor $1 + z$ equals the proportion of the two scalefactors of two different times and therefore it says something about how much the universe expanded between this two times. Back to the problem. One now knows the size of the part of our present universe which might have been in thermal equilibrium. To get the wanted angle one must divide this by the distance the light has traveled from the moment of decoupling till now. This distance is equal to $c(t_0 - t_{dec})$. Use the small angle approximation $sin(\theta) \approx \theta$ then one gets

$$\theta \sim \frac{2ct_{dec}(1 + z_{dec})}{c\,(t_0 - t_{dec})} \sim 2^\circ \tag{52}$$

For t_0 one filled in $t_0 = 1.410^{10}yr$ and we also used $z_{dec} = 1100$. The redshift can be calculated by combining equation (14) and (20)

$$1 + z_{dec} = \frac{a_0}{a_{dec}} = \frac{T_{dec}}{T_0} \tag{53}$$

and filling in $T_0 = 2.725K$ and $T_{dec} = 3000K$ [3, 4]

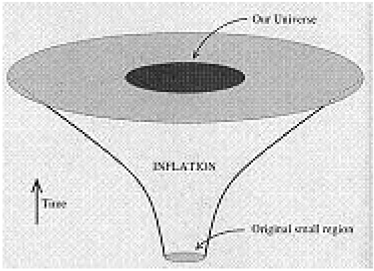

Figure 3: *Inflation* [3]

According to the Big Bang Cosmology all parts of the universe should have been in thermal equilibrium so the fact that only parts of the universe which are separated by at most two degrees could have been in thermal equilibrium is a big problem! The solution for this horizon problem and some other problems is the so called inflation proposed by Alan Guth in 1981. Inflation is a period in the evolution of the universe in which during which the scalefactor was accelerating, with other words an accelerating expansion of the universe. A part of universe sufficiently small to be in thermal equilibrium, will be much greater after inflation - much greater than the universe one can observe today, see also figure (3) [3].

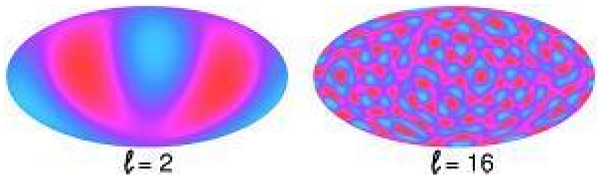

Figure 4: *Two all-sky maps of the anisotopy, measured at different angles* [21]

2 The anisotropies of the CMB

2.1 The spectrum of the anisotropies

The CMB is almost isotropic, which is one of the most important observations which accounts for the cosmological principle [6]. In fact the CMB is not completely isotropic, but contains small fluctuations in the temperature. This has been measured - one will come back to this later in the section about experiments - and gives pictures like figure (4). To make such pictures the difference in temperature is measured for a certain angle. The smaller the angle, the more precise the picture one gets, as can be seen in figure (4).

However, the interesting way of picturing the anisotropies is by making an angular power spectrum [7]. The temperature anisotropy is defined the following way:

$$\frac{\triangle T}{T}(\theta,\phi) = \frac{T(\theta,\phi) - \bar{T}}{\bar{T}} \tag{54}$$

An expansion in spherical harmonics can be made, this gives

$$\frac{\triangle T}{T}(\theta,\phi) = \sum_{l=1}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_m^l(\theta,\phi) \tag{55}$$

This expansion in spherical harmonics is the analogue of a plane-wave Fourier expansion for the surface of a sphere. Like the plane-wave modes are a complete orthonormal set in a flat three-dimensional space, the Y_m^l's are a complete orthonormal set of functions on a sphere. One can also use another notation. In this case one writes the temperature field in the universe as:

$$T(\vec{x},\hat{p},\eta) = T(\eta)\left[1 + \Theta(\vec{x},\hat{p},\eta)\right] \tag{56}$$

and than expanding Θ in spherical harmonics gives:

$$\Theta(\vec{x},\hat{p},\eta) = \sum_{l=1}^{\infty}\sum_{m=-l}^{l} a_{lm}(\vec{x},\eta) Y_m^l(\hat{p}) \tag{57}$$

where $\vec{x}$ is the position vector, $\hat{p}$ the direction of the incoming photons and η the conformal time (the distance light could have traveled since $t = 0$).

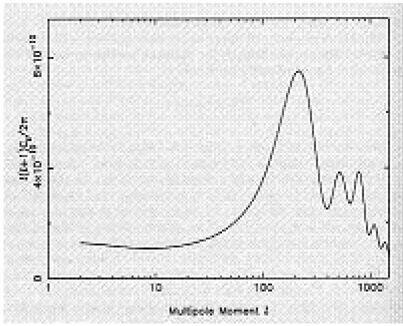

Figure 5: *Angular power spectrum as predicted by computersimulations* [3]

The definition of the conformal time is the following:

$$\eta = \int_0^t \frac{dt'}{a(t')} \tag{58}$$

To make the angular power spectrum we define

$$C_l =< |a_{lm}|^2 > \tag{59}$$

The statistical properties cannot depend on the m index, because the coordinates are rotational invariant. The average is a statistical average taken over an ensemble of all possible skies. In measurements one can only observe the CMB from a special place in the universe. Therefore one uses an ergodic hypothesis: an average over the probability ensemble is the same as an average over all spatial positions within a given realisation. For small angular scales it is possible to average over many different pairs of directions within the same angle. However, for large scales this is very difficult and large scale measurement are therefore dominated by the effect of cosmic variance. Cosmic variance is the difference between our region of the universe compared to the average region of the universe. For the angular power spectrum the quantity on the vertical axis is $\frac{l(l+1)C_l}{2\pi}$ and the quantity on the horzontal axis is the multipole moment l. This spectrum can be calculated numerically and will give something like the figure above, figure (5). Explanation of the physical meaning of the spectrum will be given in the next sections.

There is an relation between the multipole moment l and the angular scale, this relationship is [3, 8, 9]

$$\theta \sim \frac{180^\circ}{l} \tag{60}$$

When an all-sky CMB anisotropy map or an angular power spectrum is made, the contribution of several effects is already removed. An example of this is the infrared radiation emitted by the Milky Way itself. However,

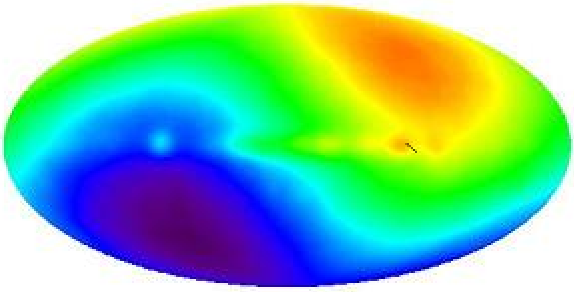

Figure 6: *Dipole as measured by COBE* [22]

the most important effect is the $l = 1$ perturbation, the dipole. For the temperature variation of the dipole holds

$$T(\theta) = T_0 \left(1 + \frac{\triangle T_D}{T_0} \times cos(\theta)\right) \tag{61}$$

It is called dipole because it has two extremes on opposite sides with a smooth transition in between. The point of largest blueshift indicates the direction in which we are heading relative to the CMB and the point largest redshift is immediately opposite, see figure (6).

The frame of the CMB coincides the frame of the Hubble flow (the frame that is at rest with respect to the movement of clusters of galaxies due to the expansion of the universe). In this frame the CMB appears isotropic and therefore there's no ϕ-dependence in equation (61). The fact that the CMB coincides to the frame of the Hubble flow is quite reasonable, because the CMB doesn't have any internal movement like planets or (clusters of) galaxies. Several shifts contribute to the Dopplershift:

- the earth orbits the sun with a velocity $v \approx 30kms^{-1}$
- the sun orbits the centre of the Milky Way with a velocity $v \approx 2220kms^{-1}$
- the Milky Way interacts with other Local Group galaxies and has also an infall into the Virgo Cluster
- the Virgo Cluster itself may have a systematic motion.

From the dipole the earth's motion through the universe can be determined, but is has no significance for considering the CMB, because it doesn't tell anything about the intrinsic properties of the CMB [8, 10].

2.2 The origin of the anisotropies

The temperature fluctuations of the CMB can be divided into primary and secondary fluctuations. The primary CMB fluctuations are the temperature fluctuations that arise from density fluctuations of the last scattering surface. The primary fluctuations contain gravitational redshift, acoustic and damping effects. From these primary fluctuations estimates of several cosmological parameters can be made, including the baryon density of the universe, the dark matter density and the curvature of the universe. They also can tell something about the still unknown mechanism which has generated the density fluctuations in the early universe. The primary CMB fluctuations form the dominant contribution to the temperature fluctuations. The secondary CMB fluctuations are generated at later times and they mainly come from interaction of CMB photons with cosmic structures. These cosmic structures were formed when the density fluctuations became large. They act at angular scales associated with the cosmic structures themselves, which are smaller than angles at which the primary fluctuations act. The secondary temperature fluctuations may provide important information about the process of structure formation and about the value of the Hubble constant [11, 12]. Besides temperature fluctuations also polarization arises.

2.2.1 Primary temperature fluctuations

Sachs-Wolfe effect The main source of temperature fluctuations on large scales is due to variations in the strength of gravity at the surface of last scattering. These gravitational potentials are generated from density fluctuations through the Poisson equation:

$$\nabla^2\phi = 4\pi G\rho, \tag{62}$$

where ϕ is the gravitational potential, ρ is the density and G is the gravitation constant. In spherical coordinates

$$\nabla^2\phi = \frac{1}{r^2}\left[\frac{\partial\left(r^2\frac{\partial\phi}{\partial r}\right)}{\partial r}\right] \tag{63}$$

Combining equations (62) and (63) and integrating twice over r gives a solution for the gravitational potential ϕ:

$$\phi = \frac{2\pi}{3}G\rho r^2 \tag{64}$$

The only natural length to use for r is the horizon distance $r_{HOR} = \frac{1}{H}$ so this will give

$$\phi = \frac{2\pi G\rho}{3H^2} \tag{65}$$

If the photons last scattered with a region where the gravitational potential is larger than average, they have lost energy, because they had to climb out of the gravitational well. Therefore the temperature of the photons from

Figure 7: *Picture of strings in potential wells and hills* [14]

such a direction is lower than average. The same way the temperature of photons that last scattered with a region where the gravitational is less than average will be higher than average. This effect is called the Sachs-Wolfe effect [4, 11, 13].

Sound waves The early universe was a hot plasma existing of nucleons, electrons and photons. The strong particle interactions (Coulomb interaction, Thomson scattering) tied the nucleons, electrons and photons together so they behave like a single fluid, the photon-baryon fluid [13, 14]. What does this have to do with sound waves? There are two forces acting on the single photon-baryon fluid. There's a gravitational force due to a difference in density and also the radiation pressure due to difference in the pressure acts on the fluid. These two forces competes with each other, the gravitational force takes care of compression, while the radiation pressure takes care of rarefraction. Consider a region of the fluid which is cool and slightly overdense. Because of the higher density, gravitational forces will pull this region of the fluid together. Because of the contraction, the fluid will be heated. This heating will produce an additional pressure and this pressure will stop the contraction and reversing it, so the fluid will be pushed outward. Because of the expansion the fluid will cool down, the pressure will be reduced and the whole process will be repeated. The oscillation is a sound wave. One can make a comparison with string, massive balls and gravity. The radiation pressure can be represented abstractly as springs. Likewise the inertia of the fluid (its 'mass', or actually its energy density) can be represented as massive balls falling under gravity. One believes that the structure of the universe is seeded by random quantum fluctuations. Because of inflation these quantum fluctuations were stretched into cosmic scales. These fluctuations in the energy density imply fluctuations in the local gravitational potential: regions of high density generate potential wells, while regions of low density generate potential hills. Note rarefraction in potential wells is the analogue of compression in potential hills. Then we get the following picture, see figure (7) [14].

One can make a very rough equation for the perturbations:

$$\frac{\partial^2 \Theta}{\partial \eta^2} + k^2 c_s^2 \Theta = F \tag{66}$$

The symbols in this equation are the wavenumber k, the sound speed c_s and F the driving force due to gravity. For the sound speed the following equation holds:

$$c_s = \sqrt{\frac{1}{3\,(1+R)}} \tag{67}$$

where R is given by

$$R = \frac{3\rho_B}{4\rho_\gamma} \tag{68}$$

The ratio $\frac{\rho_B}{\rho_\gamma}$ is the ratio of baryons to photons [9]. Equation (66) is similar the equation for the simple harmonic oscillator with mass m, force constant k and an external force F_0:

$$\frac{d^2x}{dt^2} + \frac{k}{m} = \frac{F_0}{m} \tag{69}$$

At the time of recombination nuclei together with free electrons formed atoms and the photons became free to travel. So at this moment the sound waves ceased oscillating. The time between the beginning of the universe and the moment of recombination is about 300.000 years and only during this period the sound waves were oscillating [10]. In these 300.000 years the sound waves have to live, they can only move a limited distance - oscillate a certain number of times - before freezing out. The distance the sound waves can travel in this 300.000 years is called the 'sound horizon'. The first and largest peak in the angular power spectrum corresponds to sound waves that had just reached maximum compression when they froze out. The successive peaks correspond to higher frequency waves which correspond to sound waves that just alternately reached maximum rarefraction and compression at the time of decoupling. The odd peaks belong to compression and the even peaks belong to rarefraction. [13, 15] Those waves that were in a state of either maximum compression of maximum rarefraction at recombination produce the most significant fluctuations of the CMB. The first peak is also called the *fundamental peak* and also *first acoustic peak* and its angular size - approximately one degree - is called the *fundamental scale*. To the left of this peak the wavelengths of the corresponding sound waves were too long to have completed even half an oscillation prior to recombination [10].

Damping There are two damping effects that occur at small scales. They have to do with the fact that the fluid is not perfect and also with the finite time recombination takes. One effect is that the coupling between photons and baryons only holds at scales larger than the diffusion length. Photons scattered by electrons through Thomson scattering perform a random walk and diffuse away. There are about a factor 10^9 more photons than electrons and so they drag the electrons with them, they in turn drag the photons by collisions. Therefore the sound waves will be damped below the diffusion scale, which implies a decrease of the temperature fluctuations. This effect is called Silk damping. The second effect comes from the fact that the CMB photons we observe were not all emitted exactly at the same moment, because recombination is not completely instantaneous. In fact it lasted for a redshift interval of around $\triangle z \sim 80$ For modes which experience several oscillations during this interval the net effect is that the fluctuations will be averaged during recombination. Because of the recombination of electrons and protons the Silk damping is enhanced. This occurs because of the rapid increase of the diffusion scale during recombination [11, 15].

2.2.2 Secondary temperature fluctuations

The secondary temperature fluctuations are generated after recombination and those come from interaction of CMB photons with cosmic structures. The secondary effects can be divided in two categories: those generated by gravity and those generated by scattering off free electrons. Several effects contribute to these secondary temperature fluctuations: the Integrated-Sachs-Wolfe effect, Rees-Sciama effect, Sunyaev&Zeldovich effect, Ostriker-Vishniac effect, gravitational lensing and Doppler effects. Of these effects only the Integrated-Sachs-Wolfe effect, the Sunyeav&Zeldovich effect and gravitational lensing will be further explored. Compared with the primary temperature fluctuations, the secondary fluctuations provide more details on the evolution of structure and less robust constraints on the background parameters [12].

Integrated Sachs-Wolfe effect A photon travelling through a time-varying gravitational potential will gain energy when the well becomes shallower and will loose energy when the well becomes deeper. Besides gravitational potentials, also gravitational waves will contribute to the CMB anisotropies. These gravitational waves are a prediction of general relativity. Just like the movement of a charged object produces electromagnetic waves, also the movement of a mass can produce waves. These waves are gravitational waves. Contrary to some electromagnetic waves, gravitational waves can intrude all matter. The velocity of these waves is the velocity of light, just like for electromagnetic waves. Object that move around each other produce gravitational waves. The larger the mass of the object, the stronger these gravitational waves are. The place where a gravitational wave appears will be wrinkled. When a gravitational wave travels through a medium the effect it will have is to shrink and extend spacetime in directions orthogonal to the direction of propagation of the wave. The interactions are very small and therefore there has not been achieved a direct detection of gravitational waves yet. However, a number of consequences of the existence of gravitational waves has been observed. For example two celestial bodies that slowly narrow their orbit and finally coalesce. When an object emits gravitational waves, it will loose energy and therefore it's motion will slow down, so the observation of two celestial bodies that slow down is in accordance with the prediction of gravitational waves. Photons that travel through a spacetime with shrinks and extend due to the gravitational waves will gain or loss energy and therefore the gravitational waves will effect the temperature fluctuations of the CMB photons [11, 16].

Gravitational lensing Gravitational lensing is an effect which follows form Einstein's Theory of General Relativity. Light will be deflected when it passes some massive object. The defection angle is small and therefore this effect arises only at small angular scales [1, 11].

Sunyaev-Zeldovich effect The Sunyaev-Zeldovich effect is an observed anisotropy in the CMB spectrum caused by scattering off the cold microwave photons against the free electrons of the hot ionized gas in clusters of galaxies. For the thermal Sunyaev-Zeldovich effect only the motion of the cluster relative to the Hubble expansion is taken into account. The kinetic Sunyaev-Zeldovich effect is a weaker effect, due to the peculiar motion of the cluster itself [17].

2.3 Determining cosmological parameters

The parameters that can be extracted from the study of fluctuations in the CMB are called *cosmological parameters*. One can distinguish two categories:

- The *astrophysical parameters* are the parameters that influence the propagation of density waves at recombination directly.
- The *primordial parameters* are indirect parameters which deal with physical processes that generated these density waves in the early universe. The processes are not well known, so therefore there's a lot of freedom in the way one chooses these parameters [11].

From the primary temperature fluctuations the following astrophysical parameters can be determined: the curvature of the universe, the baryon density of the universe and the matter density. From the secondary temperature fluctuations the Hubble constant can be determined [18].

2.3.1 The curvature of the universe

The geometry of the universe can be determined by the angles at which the acoustic peaks occur. Call the wavelength of the first acoustic peak λ_f. This wavelength can be computed from the physics of the early universe, where one uses the fact that the sound speed in a hot relativistic plasma is about 60 percent of the speed of light. The relationship between the angular size and the wavelength is given by

$$\theta = \frac{\lambda_f}{d}, \tag{70}$$

where d is the distance to the surface of last scattering. The angular size we observe is a function of both distance and of the spatial geometry of the universe [10, 13]. To understand this one needs to consider the geometry of space. In Euclidean geometry the angular size θ - the angle occupied upon the sky - of a object with known length l can be given the formula

$$\theta = \frac{l}{d}, \tag{71}$$

where d is the distance to the object. If we use this formula in the case of the first acoustic peak the wavelength is the length of the object and the distance d to the object is the sonic horizon and together with formula (71) this gives formula (70). The value of the angular size depends on the geometry of the universe. In particular the sum of the interior angles of a triangle depends upon symmetry, in flat geometry this relationship is given by (70). In hyperbolic geometry (open universe) the angle would be smaller than this - the peaks will shift to the right and in spherical symmetry (closed universe) the angle would be larger - the peaks will shift to the left [10].

2.3.2 The baryon density

The baryon density will effect the relative heights of the peaks. A larger baryon density will take care of a larger gravitational force with respect to the pressure. The peaks that correspond to compression of the sound waves (the odd peaks) will therefore be higher and the peaks that correspond to rarefraction of the sound wave (the even peaks) will be lower. An other effect of the baryon density is on the damping. An increased baryon density will decrease the diffusion length and therefore damping will move to smaller scale angles [9]. From formula (66) one can see a third effect of the baryon density. A larger baryon density will reduce the sound speed, because baryons are heavy. The effect of this larger density will then be a decrease in the frequency of the oscillations, so the peaks will shift and the spacing between the peaks gets larger [14].

2.3.3 The dark matter density

The series acoustic peaks is sensitive to the energy density ratio of dark matter to radiation in the universe. The absolute dark matter density can be calculated, because the amount of radiation is known, recall formula (13). One expects there to be an distinction between modes that started oscillating when the universe was radiation dominated and those that started oscillating when the universe was matter dominated. The difference is due to the fact that the gravitational potential wells and hills are created by differences in the energy density. In the case of an radiation dominated universe, the potential wells and hills are created by differences in the radiation density, likewise in the case of a matter dominated universe, these wells and hills are created by differences in the matter density. Consider the case of a radiation dominated universe. Something happened when the fluid was in its most compressed state. At this time the density fluctuation stabilized, leaving the gravitational potential to decay with the expansion of the universe. The fluid then saw no gravitational potential to fight against as it bounced back and therefore the amplitude of the oscillations increases. This driving effect does not come into play when the universe was dominated by dark matter. So that is were the distinction between modes that started oscillating when the universe was radiation dominated and those that started oscillating when the universe was matter dominated comes from. The universe was radiation dominated only in its earliest epochs, because the radiation density decreases faster than the matter density during the expansion of the universe, compare formula (10) and (11). Besides modes of smaller wavelength start to oscillate first. Because of these two reasons only the small scale modes (the lower acoustic peaks) are effected by the driving effect. So one expects an increase of the amplitude of the peaks going from low multipoles (high angles) to high multipoles (low angles). This transition contains information about the ratio of dark matter and radiation density [14].

2.3.4 The Hubble constant

In 1929 Hubble did the important observation that galaxies are moving away from earth with a velocity proportional to the distance to earth. The Hubble's law is given by

$$\vec{v} = H\vec{r} \tag{72}$$

where H is the Hubble constant. The Hubble constant can be extracted from the thermal Sunyaev-Zeldovich effect. The size of the galaxy cluster can be estimated by measuring how the spectrum of the CMB has changed due to the thermal Sunyaev-Zeldovich effect. From both the linear size and the angular size at which it appears to the observer one can use trigonometry to determine the distance from the observer to the cluster. When also the redshift is measured, one can calibrate the Hubble constant using formula (72).

In reality this way to measure the Hubble constant is very difficult, because the way the CMB photons will scatter depends on the density and therefore the distribution of hot gas in the cluster should be known. This distribution can be roughly measured from the Bremsstrahlung emitted by the cluster [3, 17].

2.4 The experiment

The CMB was accidentally discovered by Penzias and Wilson in 1965. After this discovery several experiments were done. Some experiments were done from the ground, other in a balloon or rocket, but the most important and secure experiments were done with satellites. There are a few reasons why experiments done from the ground wouldn't be that secure. Besides the problem of external sources of radiation on the earth, there's also another problem with ground experiments. The frequency of the CMB lies in a region of frequencies that is blocked by the water molecules in the atmospheres. So experiments should be done outside the atmosphere of the earth. For nearly 25 years several balloons and rockets were send to the top of the atmosphere. The balloon and rocket data indicated that the CMB had the same temperature at all points in the sky, whith in the experimental errors [10].

In 1989 the Cosmic Background Explorer (COBE) was launched. The COBE existed of three different experiments, with as the leader of the complete mission John Mather. The first experiment, Diffuse Infrared Radiation Background Experiment (DIRBE) was going to look at the redshifted light emitted by the first generation of stars. The second experiment, Far Infrared Absolute Spectrophometer (FIRAS) was meant to make a very precise measurement of the CMB-spectrum to be able to conclude wether it fitted a black body spectrum or not. At last there was done a third experiment, Differential Microwave Radiometer (DMR) to look at the fluctuations of the CMB. The cosmic background radiation was found to obey a perfect black-body law to better than 0.03%. Such a precision was never reached before. The temperature of the CMB was confirmed to be $2.725K \pm 0.002K$. The DMR simultaneously compared the radiation coming from two directions at three different frequencies. DMR mapped the full sky and found that the temperature of the CMB was nearly the same in all directions. This was already found in earlier experiment, but not with such a level of precision. However, DMR found very small anisotropies, near the level of sensitivity of the instrument, less than about one part in 10^5. COBE could only look at scales of at least $7°$. So COBE established the existence of temperature fluctuations, but a more secure measurement of these fluctuations was needed [2, 10]. In 2000 the results of the BOOMERanG experiment were published.

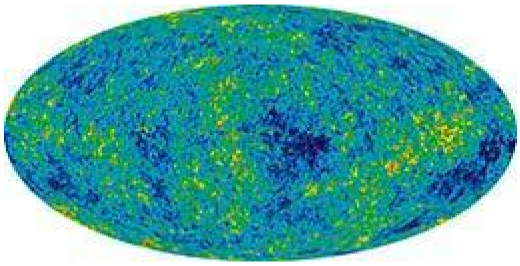

Figure 8: *All-sky CMB anisotropy map, measured by WMAP* [23]

BOOMERang stands for 'balloon observations of millimetric extragalactic radiation and geophysics'. BOOMERang mapped the CMB anisotropies over only a limited region of sky, but did so with a high sensitivity and an angular resolution of better than 1°. BOOMERanG measured about ten days continuously from an altitude of 35 km above Antartica [7]. In 2001 the Wilkinson Microwave Anisotropy Probe (WMAP) was launched. Originally it was named MAP, the W was put in front in honour of Wilkinson after his dead, an important astrophysics who had done a lot of work with the experiments of the CMB [2].

This satellite was placed at one of the Earth-Sun Lagrangian points known as L2. These Lagrangian points are points where objects tend to hover stably with respect to the Sun and the Earth. There are five Earth-Sun Langrangian points. COBE stayed pretty close to the earth in a polar orbit, with as disadvantage the restricted band of the sky it could usefully observe, because of radiation from the sun. WMAP, the successor to COBE didn't have this disadvantage and produced an all-sky anisotropy map with an angular resolution of about 0.1°, thus this satellite had a much better accuracy than COBE. Therefore WMAP was able to measure the temperature fluctuations more detailed [7]. See (8) for a picture of the all-sky map as measured by WMAP. The data of all the experiments can be combined to create a concordance model, the current best measurement of the parameters of the universe. With a computerprogram, for example CMBFAST, several cosmological models can be put in and then a detailed prediction of the angular power spectrum is made. According to the concordance model the following values can be assigned for the various parameters:
Total density parameter $\Omega_{tot} = 1.02 \pm 0.05$
Total matter contribution $\Omega_m = 0.30 \pm 0.05$
Baryon density contribution $\Omega_B = 0.05 \pm 0.01$
Consider the value of the total closure parameter depends on the curvature of the universe according to

$$\Omega = \frac{\rho}{\rho_c} = 1 + \frac{Kc^2}{(H_0 a_0))^2} \tag{73}$$

In this equation ρ_c represents the critical density, for a flat universe with $K = 0$, so this density can be calculated by setting $K = 0$ in the Friedmann equation (1). Further symbols used are H_0 which is the current value of the Hubble parameter, a_0 the present value of the scale factor and K the curvature of the universe. In the case of a flat universe $K = 0$ and then $\Omega = 1$. This total density parameter Ω is the sum of various contributions:

$$\Omega = \Omega_r + \Omega_m + \Omega_v \tag{74}$$

where Ω_r, Ω_m and Ω_v are the density parameters of radiation, matter and vacuum. The measured value for the total density parameter almost equals one, which means we live in an almost flat universe! Such a model is predicted by inflationary models. One can see that the total matter density is not equal to the total density parameter. About 70% of our universe consists of so called dark energy. Note that the density of the radiation is in the order of 10^{-5} and therefore it is only a small fraction of the total density, so there indeed must be a amount of about 70% dark energy. The contribution of the baryon density can be obtained from the angular power spectrum of the CMB in more than one way: from the acoustic peaks and from the damping tail. The value obtained from the CMB can be compared with the value obtained from the (less precise) value from Big Bang nucleonsynthesis ($\Omega_B \approx 0.05$). Also the baryon density accounts for only a small fraction of the total matter density, most of the matter is dark matter. So we live in a flat universe that is dominated by dark energy and dark matter [2, 3, 4]. The Planck satellite will be launched in 2007 as the successor of COBE and WMAP. Its goal is to get the highest precision, highest resolution and cleanest maps. Therefore it might give the best estimates of cosmological parameters. Planck might for example be able to detect gravity waves and be able to study the Sunyaev-Zeldovich effect [19].

2.5 Conclusion

The discovery of the CMB has been a great step forwards in cosmology. The CMB has found its origin very early in the evolution of the universe; decoupling took place about 300.000 years after the Big Bang. By the Big Bang model it has been predicted that the spectrum of the CMB should be a black body spectrum. Indeed measurements, especially those done by COBE, confirmed this. But the CMB is not only important as confirmation of the Big Bang model. From the CMB one can obtain information about cosmological parameters with a precision never reached before. Most of the information can be obtained from the acoustic waves, which were caused by fluctuations in the density in the early universe. From these peaks one can deduce the curvature of the universe, the baryon density and the dark matter density. When Planck will be launched in 2007 even more secure information about the parameters of the universe might be obtained.

2.6 Acknowledgement

My supervisor for this bachelor thesis is Prof.Dr.E.A.Bergshoeff and I want to thank him for his support.

References

[1] Simon Singh, *De oerknal*, Amsterdam, Uitgeverij de Arbeidspers (2005)

[2] Michael D.Lemonick, *Echo of the Big Bang*, New Jersey, Princeton University Press (2003)

[3] Andrew Liddle, *An Introduction to Modern Cosmology*, second edition, UK, University of Sussex (2004)

[4] Donald Perkins, *Particle Astrophysics*, Oxford University Press (2003)

[5] Eugene Hecht, *Optics*, second edition, Aldephi University (1987)

[6] Ray D'Inverno, *Introducing Einstein's relativity*, Oxford, Clarendon Press (1992)

[7] edited by Mark H.Jones and Robert J.A.Lambourne, *An Introduction to Galaxies and Cosmology*, co-published edition, The Open University, Cambridge University Press (2004)

[8] P. Coles and F. Lucchin, *Cosmology: the origin and evolution of cosmic structure*, John Wiley & Sons (1995)

[9] Scott Dodelson, *Modern Cosmology*, Elsevier Science (2003

[10] John F. Hawley and Katherine A. Holcomb, *Foundations of Moderns Cosmology*, second edition, University of Virginia, Oxford University Press (2005)

[11] http://www.planck.fr (31-05-06)

[12] http://arxiv.org/PS_cache/astro-ph/pdf/9604/9604166.pdf (1-06-06)

[13] http://map.gsfc.nasa.gov/m_mm/sag_parameters3.html (31-05-06)

[14] http://background.uchicago.edu/w̃hu/intermediate/intermediate.html (22-06-2006)

[15] http://arxiv.org/PS_cache/astro-ph/pdf/0401/0401108.pdf (1-06-06)

[16] Kris Verburgh, *Schitterend! Over het universum*, third edition, Pandora, Amstel Uitgevers BV (2003, 2005)

[17] http://www.everything2.com/index.pl?node_id=1408837 (15-06-2006)

[18] http://arxiv.org/PS_cache/astro-ph/pdf/9512/9512139.pdf (1-06-2006)

[19] http://aether.lbl.gov/www/projects/cosa/#GOALS (22-06-2006

[20] http://www.bell-labs.com/project/feature/archives/cosmology/ (22-06-2006)

[21] http://www.astro.ucla.edu/w̃right/CMB-DT.html (22-06-2006)

[22] http://antwrp.gsfc.nasa.gov/apod/ap010128.html (22-06-2006)

[23] http://map.gsfc.nasa.gov/m_or.html (22-06-2006)