Quantum Mechanics and Retrocausality

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Abstract. The classical electrodynamics of point charges can be made finite by the introduction of effects that temporally precede their causes. The idea of retrocausality is also inherent in the Feynman propagators of quantum electrodynamics. The notion allows a new understanding of the violation of the Bell inequalities, and of the world view revealed by quantum mechanics.

1. Introduction

Dirac was never happy with quantum electrodynamics, although it was in large part his own creation. In old age, during an after-dinner seminar in 1970 that I attended in Austin, Texas, he lambasted such upstarts as Feynman, Schwinger, Tomonaga, and their ilk, under the dismissive collective term 'people'. These "People neglect infinities in an arbitrary way. This is not sensible mathematics. Sensible mathematics involves neglecting a quantity when it is small — not neglecting it just because it is infinitely great and you do not want it." A timorous spirit among the chastened listeners asked: "But, Professor Dirac, what about $g-2$?", referring of course to the g-factor in the expression for the magnetic moment of the electron. Dirac's own equation had predicted that this factor should be precisely 2, and the highly accurate quantum electrodynamical calculation of its deviation from 2 was, and is, one of the tours de force of modern physics. The agreement with painstaking experimental measurement of this quantity is phenomenal (the Particle Data Group gives on the World Wide Web ten digits of agreement after the decimal point[1]). But the old maestro had his own views about this remarkable result: "It might just be a coincidence," he remarked evenly.

Quantum mechanics, married to electromagnetism, has produced a very successful theory, as measured by its empirical adequacy. The matter is not so adequate, however, at a conceptual level. There are still many competing *interpretations* of what quantum mechanics is telling us about the nature of the world. Despite the early preoccupation with the breakdown of determinism, the serious difficulties have to do rather with causality, which is by no means the same thing. Classical electromagnetic theory is in fact not immune to such problems either: the only known way to remove disastrous infinities in the theory of point charges interacting through the electromagnetic field is by the introduction of retrocausal effects. Quantum electrodynamics inherits the diseases of causality and of divergence from both of its parents. Their nature is pervasive, the cure unknown.

2. Advanced Potentials

An electrically neutral particle, of mass m, subject to a force $\vec{F}$, satisfies Newton's second law of motion, which may be expressed in the form

$$m\vec{a} = \vec{F}\,, \tag{1}$$

where $\vec{a} = \ddot{\vec{r}}$ is the acceleration, on condition that $|\dot{\vec{r}}| << c$, so that relativistic corrections may be neglected. A similar charged particle cannot satisfy the same equation, because an accelerated charge emits electromagnetic waves, losing energy in the process. Newton's law may be repaired by adding an effective radiative damping force that accounts for this extra source of energy loss to space:

$$m\vec{a} = \vec{F} + \vec{F}_{\rm rad}\,, \tag{2}$$

where one finds, for a point charge e,

$$\vec{F}_{\rm rad} = \frac{2e^2}{3c^3}\dot{\vec{a}}\,. \tag{3}$$

We may rewrite Eq.(2)-(3) in the form

$$m(\vec{a} - \tau\dot{\vec{a}}) = \vec{F}\,, \tag{4}$$

where

$$\tau = \frac{2e^2}{3mc^3}\,,$$

is called the Abraham-Lorentz relaxation time. For an electron it is about 6×10^{-24} sec., in which time light travels only about 10^{-13} cm., the size of a proton.

The general solution of Eq.(4) is

$$\vec{a}(t) = \frac{1}{m\tau}\int_t^c dt'\, \mathrm{e}^{(t-t')/\tau}\vec{F}(t')\,,$$

where c is an integration constant. Clearly $\vec{a}(t)$ blows up exponentially as $t \to \infty$, the so-called runaway solution, unless $c = \infty$. Accordingly, we choose this latter value, and find we can rewrite the solution in the form

$$m\vec{a}(t) = \int_0^\infty ds\, \mathrm{e}^{-s} \vec{F}(t + \tau s)\,, \tag{5}$$

from which we derive the following Taylor series in τ:

$$m\vec{a}(t) = \sum_{n=0}^{\infty} \tau^n \vec{F}^{(n)}(t)\,. \tag{6}$$

The Newton law Eq.(1), as it applies to a neutral particle, corresponds to the zeroth term only. From Eq.(5), the acceleration at time t is determined not only by the value of the applied force at time t, but also by the force at all times *later* than t.

For a simple force, one can evaluate Eq.(5) explicitly. For example, if a force is turned on at time $t = 0$, after which it remains constant, i.e. $\vec{F}(t) = 0$ for $t < 0$ and $\vec{F}(t) = \vec{K}$ for $t \geq 0$, then we find $m\vec{a}(t) = \vec{K}$ for $t \geq 0$, as we would for a neutral particle, but surprisingly $m\vec{a}(t) = \vec{K}\,\mathrm{e}^{t/\tau}$ for $t < 0$. This preacceleration violates a naïve notion of causality, according to which a cause *precedes* its effect, whereas here the force, which is not applied before time $t = 0$, produces (has already produced!) an acceleration *before* $t = 0$.

Consider next a universe consisting of many particles, at positions $x_a, x_b, \ldots$ with masses $m_a, m_b, \ldots$ and charges $e_a, e_b, \ldots$ For particle a, the relativistic generalization of Eq.(2) for the four-momentum p_a^μ is

$$\frac{dp_a^\mu}{d\tau_a} = e_a \left[F^\mu{}_\nu + R^\mu{}_\nu \right] \frac{dx_a^\nu}{d\tau_a}\,. \tag{7}$$

Here τ_a is the proper time of particle a, and $F^\mu{}_\nu$ is the retarded field tensor that gives rise to the usual Lorentz force. It may be written

$$F^\mu{}_\nu = \sum_{b \neq a} F_b^{\mathrm{ret}\,\mu}{}_\nu\,,$$

where the sum is over all the contributions to the field from the particles *other* than a itself: there is no self-interaction. The term $R^\mu{}_\nu$ is the radiation damping tensor: it corresponds to $\vec{F}_{\mathrm{rad}}$ in the nonrelativistic approximation (3). Dirac deduced the explicit form of this tensor and showed that it can be written

$$R^\mu{}_\nu = \tfrac{1}{2} \left[F_a^{\mathrm{ret}\,\mu}{}_\nu - F_a^{\mathrm{adv}\,\mu}{}_\nu \right]\,. \tag{8}$$

It is very interesting that this expression involves the advanced, as well as the retarded fields arising from particle a. For the point particles that we are considering, these fields are separately singular on the world-line of a itself, but their difference (8) is finite.

To simplify the notation, we will henceforth suppress the Lorentz indices. It is important to distinguish the sum $\sum_{b\neq a}$, in which one sums over all particles *except* a, in order to calculate the influence of the rest of the universe on particle a, and the sum $\sum_b$, in which a is also included, giving a quantity that refers to the universe in its entirety.

$$\begin{aligned} F+R &= \sum_{b\neq a} F_b^{\mathrm{ret}} + \tfrac{1}{2}\left[F_a^{\mathrm{ret}} - F_a^{\mathrm{adv}}\right] \\ &= \sum_b F_b^{\mathrm{ret}} - F_a^{\mathrm{ret}} + \tfrac{1}{2}\left[F_a^{\mathrm{ret}} - F_a^{\mathrm{adv}}\right] \\ &= \sum_b F_b^{\mathrm{ret}} - \tfrac{1}{2}\left[F_a^{\mathrm{ret}} + F_a^{\mathrm{adv}}\right] \end{aligned} \tag{9}$$

The essential assumption of Wheeler and Feynman[2] is that the universe is a perfect absorber: all radiation is absorbed somewhere and none escapes to infinity. Since a radiation field is of order $1/r$ for large distances r, to eliminate energy loss by radiation it is enough to require

$$\sum_b F_b^{\mathrm{ret}} = o\left(r^{-1}\right) \qquad\qquad \sum_b F_b^{\mathrm{adv}} = o\left(r^{-1}\right) ,$$

for all times, i.e. the sum of all retarded (advanced) fields is assumed always to vanish faster than $1/r$ at spatial infinity. However, $\sum_b F_b^{\mathrm{ret}}$ and $\sum_b F_b^{\mathrm{adv}}$ each satisfies Maxwell equations with the same sources and sinks (the charges). They are indeed two independent solutions of the same second-order equations. Hence their difference,

$$\sum_b \left[F_b^{\mathrm{ret}} - F_b^{\mathrm{adv}}\right] , \tag{10}$$

satisfies a homogeneous system of equations, i.e. a system without sources or sinks. Such a system possesses nontrivial solutions, but they are radiation fields that decrease like r^{-1} at spatial infinity: there are no $o\left(r^{-1}\right)$ nontrivial solutions. Thus the difference (10) is not merely zero at spatial infinity, it must be identically zero everywhere. Hence

$$\sum_b F_b^{\mathrm{ret}} = \sum_b F_b^{\mathrm{adv}} = \tfrac{1}{2}\sum_b \left[F_b^{\mathrm{ret}} + F_b^{\mathrm{adv}}\right] , \tag{11}$$

for all times.

On combining this result with Eq.(9), we obtain

$$\begin{aligned} F+R &= \tfrac{1}{2}\sum_b \left[F_b^{\mathrm{ret}} + F_b^{\mathrm{adv}}\right] - \tfrac{1}{2}\left[F_a^{\mathrm{ret}} + F_a^{\mathrm{adv}}\right] \\ &= \tfrac{1}{2}\sum_{b\neq a} \left[F_b^{\mathrm{ret}} + F_b^{\mathrm{adv}}\right] \end{aligned} \tag{12}$$

This is a stunning result: it says that to calculate the response of a charged particle to all the other charged particles in the universe, one has to sum over the fields emanating from all those other particles, *on condition that one uses the time-symmetric solution of the Maxwell equation.* In this approach there is no need, nor room, to add a further radiation damping term: it is all contained in the average of the retarded and advanced solutions of Maxwell's equations. Turning the argument around, one can say that the time-symmetric form is equivalent to, and so validates, the conventional calculation in which a retarded solution is supplemented, in a somewhat *ad hoc* manner, by a radiation damping field.

It must not be thought that we have hereby forged an arrow of time from a time-symmetric theory. This can be seen by complementing Eq.(9) by

$$\begin{aligned} F + R &= \sum_b F_b^{\mathrm{adv}} - \tfrac{1}{2}\left[F_a^{\mathrm{ret}} + F_a^{\mathrm{adv}}\right] \\ &= \sum_{b \neq a} F_b^{\mathrm{adv}} + \tfrac{1}{2}\left[F_a^{\mathrm{adv}} - F_a^{\mathrm{ret}}\right] . \end{aligned} \tag{13}$$

This is an equally valid *modus operandi*, involving the full advanced potential, supplemented by a radiation damping term, but since it is precisely minus the corresponding term in the first line of Eq.(9), it might better be called a radiation boosting term.

3. Bell Inequality

Let us turn now to the Einstein-Podolsky-Rosen scenario[3] in its modern experimental avatar[4]. We will see that the violation of the Bell inequality loses much of its impact once we entertain the notion of advanced fields.

Briefly, two photons are prepared with opposed spins by the sequential decay of a calcium atom from an excited S state, through an intermediate P state, to the ground state, which is also S. The state of linear polarization of one photon is measured by means of a birefringent calcite crystal and a photo-detector at location A, and that of the other photon by a similar arrangement at location B. The separation of A and B is several metres, and the measurement events are contained within small space-time hypervolumes that have a mutual spacelike separation. Thus the measurement events at A and B are independent of one another in the sense that no information about the result of the measurement at A can be transmitted to B in time to influence the result of the measurement there (and vice versa). This is true *only if we limit ourselves to the usual retarded fields.* The two photons are not independent, however, in the sense that their spins are correlated because of their common genesis in an atomic decay. The polarizations have, in the locution of Reichenbach, a common cause[5].

If the optical axes of the calcite crystals at A and B are parallel, then whenever a photon at A is found to go in the direction of the ordinary ray, the same is found at B. Similarly, there is perfect correlation in the case that the photons are deflected along the extraordinary ray directions. The more general situation, in which the optical axis at A is at an angle α to the vertical, and the optical axis at B is at an angle β to the vertical, leads to the following joint probabilities:

$$\begin{aligned} P_{oo}(\alpha,\beta) &= \tfrac{1}{2}\cos^2(\alpha-\beta) \ = P_{ee}(\alpha,\beta) \\ P_{oe}(\alpha,\beta) &= \tfrac{1}{2}\sin^2(\alpha-\beta) \ = P_{eo}(\alpha,\beta)\,. \end{aligned} \tag{14}$$

Here P_{oo} is the probability that the photons at A and B both go into the ordinary rays, P_{ee} that both photons go into the extraordinary rays, P_{oe} is the probability that the photon at A goes into the ordinary ray but the photon at B goes into the extraordinary ray, and finally P_{eo} is the probability that the photon at A goes into the extraordinary ray but the photon at B goes into the ordinary ray. The results Eq.(14) are predicted by quantum mechanics and confirmed by experiment.

The correlation coefficient is defined as follows:

$$C(\alpha,\beta) = P_{oo}(\alpha,\beta) + P_{ee}(\alpha,\beta) - P_{eo}(\alpha,\beta) - P_{oe}(\alpha,\beta) = \cos 2(\alpha-\beta)\,. \tag{15}$$

If we suppose, with Bell[6], that the joint probabilities, and hence the correlation coefficient, are separable, in the sense of classical probability theory, then we can write, for this correlation coefficient,

$$C(\alpha,\beta) = \sum_\lambda \rho(\lambda) C(\alpha|\lambda) C(\beta|\lambda)\,, \tag{16}$$

where λ are *hidden variables* that account for the correlations between the two photon polarizations: they arise from the birth of the twin photons in the de-exciting calcium atom. The weight $\rho(\lambda)$ is supposed to be positive and normalized; and $C(\alpha|\lambda)$ is the correlation coefficient

$$C(\alpha|\lambda) = P_o(\alpha|\lambda) - P_e(\alpha|\lambda)$$

at location A, *conditioned by the hidden variable* λ. Similarly, $C(\beta|\lambda)$ is the conditional correlation coefficient at location B. Clearly each conditional correlation coefficient, being the difference between two probabilities, lies in the interval $[-1,1]$.

The Bell coefficient is defined as the following combination of four correlation coefficients:

$$B = C(\alpha, \beta) + C(\alpha', \beta) + C(\alpha', \beta') - C(\alpha, \beta')\,. \tag{17}$$

It can be measured by combining the results of four separate runs of the experiment, with a choice of two possible orientations (α or α') of the calcite optical axis at A, and two possible orientations (β or β') at B. One can show, under the assumption of separability, and

$$\sum_\lambda \rho(\lambda) = 1\,, \tag{18}$$

with $\rho(\lambda) \geq 0$, that

$$|B| \leq 2\,. \tag{19}$$

However, by choosing the angles α, β, α' and β' suitably, one can arrange that quantum mechanics yields $B = 2\sqrt{2} > 2$. However,

$$C(\alpha, \beta) = \cos 2\alpha \cos 2\beta + \sin 2\alpha \sin 2\beta\,,$$

so the normalization Eq.(18) is ruined[1] — on the right-hand side of Eq.(18) we obtain 2 instead of 1! We must conclude that something is amiss; and we seem to have (at least) the following options:

1. No hidden variables can be found that screen off the common cause.
2. Classical probability theory is simply inapplicable in the quantum domain, in particular Kolmogorov's definition of stochastic independence is inappropriate[7].
3. Advanced as well as retarded fields are present.

In this paper we will concentrate on the third possibility. If the absorption of the photon at A, after its passage through the calcite crystal at A, is accompanied by an advanced, as well as a retarded field, then *information about the interaction of the photon at A, in particular details about the polarizer orientation at the moment of measurement, will ride the advanced wave back to the genesis of the photon pair, arriving at the calcium atom just at the moment that it de-excites.* We can understand how, even if the orientation of the A polarizer is changed at the last moment before the polarization measurement, still the interaction can carry information back about the measurement configuration. This way of speaking about information being carried back and forth, as if there were a sort of internal biological time of the sort that science

[1] $\rho(\lambda) = 1$, $\lambda = \{1, 2\}$. $C(\gamma|1) = \cos 2\gamma$, $C(\gamma|2) = \sin 2\gamma$, $\gamma = \{\alpha, \beta\}$.

fictional time travellers seem to carry about with themselves, is imprecise and may be confusing. It is better to say that, in the advanced field approach, one has a self-consistent picture in which the state of the photon's polarization is correlated to its future, as well as to its past interactions. The notions of 'cause' and 'information' are replaced by that of 'correlation'.

In one variant of Aspect's experiment, the selection between the angles α and α' at A, and β and β' at B, was changed randomly by two independent oscillators every few nanoseconds. Still the predictions of quantum mechanics were borne out and the Bell inequality violated. Most people interpret this as a demonstration of nonlocality (more soberly of nonseparability). With option 3 we can retain Lorentz covariance while achieving action at a distance. Is this action local or nonlocal? In a sense it is a semantic matter. It is not usual to call conventional retarded field theory nonlocal, the idea being that a particle is only influenced by a distant causal agent in the particle's past light cone. This influence is fleshed out by imputing a real existence to the field (in quantum theory to the field quanta). In this way the field serves as a messenger from afar, bringing influence and information at no more than light speed and delivering it in the vicinity of the particle. One might describe advanced action also as being local in an analogous manner: an influence is transmitted by the advanced field, also within the light cone, arriving in the vicinity of the particle to deliver its information, much on a par with the retarded case. However, this account, even after deanthropomorphization in terms of correlations rather than of causes and of influences, is incomplete. Since correlations can be established forwards and backwards in time, really the only logical requirement is one of consistency. The theory need only be such that it is impossible for an event in a space-time hypervolume both to occur and not to occur[2].

4. Retrocausality

According to David Hume, causality is based on nothing more than the observed constant conjunction of two or more kinds of events, say A and B. It is a mere habit we have to call the earlier of the occurrences, say A, the cause, and the later, B, the effect; no relation of necessity, nor even of likelihood, of a B's succeeding an A in the future can be deduced. If we replace the word 'habit' by 'theory', then we may reconstrue Hume's admonition as the trite Scottish verity that we have

[2] We leave out of consideration the science fiction scenario of many worlds. This option is logically flabby and it carries moreover an unwieldly metaphysical baggage.

no proof that a theory, based on the results of observations in the past, will yield reliable predictions in the future, no matter how numerous the observations in question are. Indeed, we neither have, nor expect to be able to provide, such a proof concerning empirical matters. Moreover, if it is a mere habit, a mere linguistic convention, to call the temporal antecedent a cause, and the successor an effect, why should we not expand our horizons, generalize our theories, and envisage causes that can occur *later* than their hypothesized effects?

In his intriguing article "Bringing About the Past", Michael Dummett has indeed claimed that the temporal asymmetry of the causal relation is contingent rather than necessary[8]. He describes two situations in which one might speak of a voluntary action performed with the intention of bringing about a past event. Nevertheless, stringent conditions must be satisfied to ensure the coherence of such a standpoint. In particular, Dummett claims that it is *incoherent* to hold all of the following claims:

1. There is a positive correlation between an agent's performing an action of type A at time t_A and the occurrence of an event of type B at time t_B, where $t_A > t_B$.
2. It is entirely within the power of the agent to perform A at time t_A, if he so chooses.
3. It is possible for the agent to find out, at time t_A, whether B has or has not already occurred, independently of his performing A.

One of the two examples that Dummett describes concerns a tribe that has the following custom: "Every second year the young men of the tribe are sent, as part of their initiation ritual, on a lion hunt: they have to prove their manhood. They travel for two days, hunt lions for two days, and spend two days on the return journey; ... While the young men are away from the village the chief performs ceremonies—dances, let us say—intended to cause the young men to act bravely. We notice that he continues to perform these dances for the whole six days that the party is away, that is to say, for two days during which the events that the dancing is supposed to influence have already taken place. Now there is generally thought to be a *special* absurdity in the idea of affecting the past, much greater than the absurdity of believing that the performance of a dance can influence the behavior of a man two days' journey away; ... " Ref.[8], pages 348-9. In physicists' terms, retrocausality seems even more absurd than action at a distance.

The chief is a wise and rational man: he believes the first of the above-mentioned three claims, at any rate as a statement of the significant statistical efficacity of his magic dancing. Let us further suppose that he does not believe that he is somehow hindered from dancing, or perhaps caused to dance inadequately, during the last two days, in the case that his young men have been cowardly. Then he must deny the third claim: he must assume that there is no way that he can find out, during the crucial days 5 and 6, what in fact has happened during days 3 and 4. For if it were possible to find it out, he could *bilk* the correlation. That is to say, he could choose to dance properly if, and only if, he knew that his men had not been brave. Then there would not be a positive correlation of the sort envisaged in claim 1.

It seems that we, as anthropologists, would at any rate accept claim 3, and thus conclude incoherence. With the aid of radio communication and a field worker, we could always arrange a bilking scenario, so that A could not count, even stochastically, as a cause of the earlier event B. But is there a situation in which claim 3 could defensibly be denied? There seem indeed to be such cases in subatomic physics. For example, the state of polarization of a photon, which has passed through one polarizer, and will pass through a second polarizer, is a property that we can only test by passing it through the next polarizer that it will encounter. If we choose to insert a calcite crystal in the path of the photon in such a way that it effects a polarization measurement, then *this crystal is the next polarizer.* If it be claimed that the state of polarization of a photon is correlated, not only with the orientation of the polarizer in its past, but also with that of the polarizer in its future trajectory, no bilking of the claim is possible. Here is indeed a clear candidate for retrocausal effects.

5. The View from Nowhen

Is there a way to fit the notion of retrocausality into a general theoretical framework, rather than merely to permit its fugitive occurrence when all bilking scenarios are impossible? The Australian philosopher Huw Price elaborates a Weltanschauung that he calls the view from nowhen[9]. His point of departure is the time reversal (T) invariance of microscopic processes[3]. When two inert gases of different colours, initially segregated and at different temperatures, are allowed to mix, the approach to an equilibrium mixture, of an intermediate colour and at an intermediate temperature, is irreversible, although the dynamics of the molecular collisions is T-invariant. A reversed video recording of the process would not look queer at the level of individual collisions, seen one by one, but it would appear odd at the macro-level, where it would

[3] This must be generalized to PCT invariance for some electroweak interactions, for example those responsible for K^0-decay.

show an apparently spontaneous segregation of the two gaseous components. It is generally agreed that the Stoßzahlansatz of Boltzmann, an example of what Price calls PI^3, or the principle of the independence of incoming influences, is not acceptable as an *explanation* of the irreversibility in question. For if PI^3 holds, why should not $PIOI$ hold, the principle of the independence of outgoing influences? If one suggests that $PIOI$ breaks down because correlations are generated by a collision, then one must ask whether after all PI^3 is justified. That is, if correlations are generated in a collision process, may they not be present *before* as well as after the scattering? There seems in fact to be no good reason for adopting a double standard in this matter. Indeed, to do so in the search for a thermodynamic arrow of time is a flagrant example of *petitio principii.*

A convincing case can be made that the the master arrow of time is cosmological, and the major task lies in explaining why the cosmos had such a low entropy in what for us is the distant past. The thermodynamic arrow follows readily: there is no need for an *ad hoc* PI^3 without a $PIOI$. The Wheeler-Feynman time symmetric treatment of electromagnetic radiation implicitly appeals ultimately to cosmology, for the effective retardation arises from the assumption of perfect future absorption. This absorption is treated as a matter of irreversible thermodynamics, in terms in fact of a phenomenological absorptive (complex) refraction index. The thermodynamic arrow is tied to the cosmological one, and Wheeler and Feynman reason that radiation appears to us to be retarded because of thermodynamic processes in the future universe. The reason for the direction of the thermodynamic arrow itself seems to lie in the statistical properties of the *early* universe, i.e. in the fact that it was in such a low entropy condition.

If the arrow of radiation ultimately derives from cosmological considerations, it would be desirable to show this directly, in terms of the properties of a cosmological model, rather than indirectly, via thermodynamics. This is precisely what Hoyle and Narlikar have done[10]. Suppose that the future is *not* a perfect absorber, but only works at efficiency f, in the sense that the reaction of the universe, on particle a, is not the full Dirac radiation damping of $\frac{1}{2}\,[F_a^{\mathrm{ret}} - F_a^{\mathrm{adv}}]$, but only f times this quantity. Analogously, suppose that the past is also not perfect as an absorber, but has efficiency p. That is, the boosting is not minus the Dirac term, but rather $-p$ times that quantity. Let us write the symmetric sum over all the fields acting on particle a as a general linear superposition of retarded and advanced contributions, each with its damping or boosting terms:

$$A\left\{\sum_{b\neq a}F_b^{\rm ret}+\tfrac{1}{2}f\left[F_a^{\rm ret}-F_a^{\rm adv}\right]\right\}+B\left\{\sum_{b\neq a}F_b^{\rm adv}-\tfrac{1}{2}p\left[F_a^{\rm ret}-F_a^{\rm adv}\right]\right\}\,, \tag{20}$$

with $A+B=1$. This leads to

$$(1-2A)\sum_b F_b^{\rm ret}+(1-2B)\sum_b F_b^{\rm adv}= \\ (1-2A+Af-Bp)F_a^{\rm ret}+(1-2B-Af+Bp)F_a^{\rm adv}\,. \tag{21}$$

The system is consistent if the coefficients of $F_a^{\rm ret}$ and $F_a^{\rm adv}$ vanish:

$$A = \frac{1-p}{2-f-p} \qquad B = \frac{1-f}{2-f-p}\,, \tag{22}$$

and this is indeed consistent with $A+B=1$.

The Hoyle-Narlikar relation Eq.(22) is interesting. Unless the past and the future are *both* fully absorbing, the values of A and B are uniquely defined. For $p<1$ and $f<1$, since neither A nor B is zero, the radiation from an accelerated charge is effectively neither retarded nor advanced, but a superposition of the two, and the radiation damping is a definite fraction of the Dirac value. The special case in which the future is a perfect, but the past an imperfect absorber, $f=1$ but $p<1$, leads to $A=1$ and $B=0$, which is the empirically satisfactory situation of effectively retarded radiation, together with the full strength Dirac radiation damping. With $p=1$ but $f<1$, on the other hand, we obtain $B=1$ and $A=0$. That is, in the situation in which the big bang acts as a perfect absorber but the future is not fully absorbing—in an open Friedmann model, for instance—one finds the unacceptable effectively advanced solution, with a radiation boosting term, i.e. minus the Dirac radiation damping. The main point to be made here is that, while the basic emission is time symmetric, the effective radiation is not symmetric if and only if $p\neq f$. That is, the radiative temporal symmetry is broken by an asymmetry in the absorptive properties of the past and future universe, in short by a cosmological asymmetry.

It seems that Feynman himself, after he had elaborated quantum electrodynamics (QED) in the form that we still use today, rejected only part of the credo of symmetric action at a distance[11]: "It was based on two assumptions:

1. Electrons act only on other electrons

2. They do so with the mean of retarded and advanced potentials

The second proposition may be correct but I wish to deny the correctness of the first." The reason given for accepting that a charged particle can interact with its own field was precisely the success of the calculation of the anomalous magnetic moment of the electron—the famous $g-2$ to which we alluded at the beginning.

The close similarity between the Wheeler-Feynman account of radiation and that given in QED—and also the crucial difference—can be appreciated by looking at the Green's functions of the theories. The electromagnetic field tensor may be expressed in terms of the four-potential, $A^\mu(x)$, by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,,$$

and the Maxwell equations can be written

$$\partial^2 A_\mu = j_\mu \,, \tag{23}$$

in the Lorentz gauges, for which $\partial_\mu A^\mu = 0$. Here j_μ is the four-current density. A solution of Eq.(23) is expressible as an integral,

$$A_\mu(x) = \int d^4y D_{\mu\nu}(x-y) j^\nu(y) \,,$$

where $D_{\mu\nu}$ is a Green's function that satisfies

$$\partial^2 D_{\mu\nu}(x) = g_{\mu\nu}\delta^4(x)$$

The relations between the different theories can be appreciated by comparing the various choices of Green's function. The standard classical choice is the retarded one:

$$D^{\rm ret}_{\mu\nu}(x) = -\frac{g_{\mu\nu}}{(2\pi)^4}\int d^4p \frac{{\rm e}^{-ipx}}{(p_0+i\epsilon)^2 - \vec{p}.\vec{p}} = \frac{g_{\mu\nu}}{2\pi}\theta(x_0)\delta(x^2) \,.$$

The $i\epsilon$ prescription means that the Green's function is to be interpreted as a distribution on a space of analytic functions: the implicit limit $\epsilon \to 0$ through positive values is equivalent to a small deformation of the k_0-integration contour in the appropriate direction. The advanced Green's function is obtained from the above by changing the sign of ϵ, which implies that $\theta(x_0)$ is replaced by $\theta(-x_0)$. The Green's function of the Wheeler-Feynman theory is

$$\begin{aligned} D^{\rm WF}_{\mu\nu}(x) &= \tfrac{1}{2}\left[D^{\rm ret}_{\mu\nu}(x) + D^{\rm adv}_{\mu\nu}(x)\right] \\ &= -\frac{g_{\mu\nu}}{(2\pi)^4}\int d^4p \, {\rm e}^{-ipx}\frac{P}{p^2} = \frac{g_{\mu\nu}}{4\pi}\delta(x^2) \,, \end{aligned} \tag{24}$$

where P means the principal value in the sense of Cauchy.

The QED Feynman propagator, defined through the vacuum expectation value of the time ordered product of two fields, is in QED

$$D^{\mathrm{F}}_{\mu\nu}(x) = -\frac{g_{\mu\nu}}{(2\pi)^4}\int d^4p\frac{\mathrm{e}^{-ipx}}{p^2+i\epsilon} = \frac{g_{\mu\nu}}{4i\pi^2}\frac{1}{x^2-i\epsilon}\,.$$

Now we can write

$$D^{\mathrm{F}}_{\mu\nu}(x) = -\frac{g_{\mu\nu}}{(2\pi)^4}\int d^4p\,\mathrm{e}^{-ipx}\left[\frac{P}{p^2} - i\pi\delta(p^2)\right]\,.$$

On comparing this with Eq.(24), we see that the Wheeler-Feynman Green's function is the real part of the Feynman Green's function. The extra piece, the imaginary part of the Feynman propagator, corresponds to the mass-shell contribution in momentum space, and has to do with the self-interaction of a charged particle that is coupled to the electromagnetic field. It guarantees the meromorphy of scattering amplitudes on the principal sheet of a suitably cut p^2-plane.

Microcausality, as it is now understood in quantum field theory, is expressed by the vanishing of (anti-)commutators of fields outside the light-cone; and this leads to analyticity of scattering amplitudes with respect to momenta. However, this new style causality is perfectly consistent with, indeed requires, retrocausality on the same footing as ordinary (Humean) causality. However, the heavy price that we must pay is the introduction of self interaction. This gives rise to divergences that are only provisionally hidden in the renormalization programme. Feynman was not satisfied with what he had achieved[11]: "I invented a better way to figure, but I hadn't fixed what I wanted to fix ... The problem was how to make the theory finite ... I wasn't satisfied at all."

Hoyle and Narlikar also add a self-action term to their quantized action at a distance theory, almost as an afterthought, and clearly against their better inclination[10]. As Dirac had done before them, they simply introduce an ultraviolet cut-off that breaks Lorentz covariance. Dirac writes, at the end of the fourth edition of his classic book, Quantum Mechanics[12]: "It would seem that we have followed as far as possible the path of logical development of the ideas of quantum mechanics as they are at present understood. The difficulties, being of a profound character, can be removed only by some drastic change in the foundations of the theory, probably a change as drastic as the passage from Bohr's orbit theory to the present quantum mechanics."

Could it be that the change to the view from nowhen, following in the footsteps of Wheeler, Feynman, Hoyle, Narlikar and Price, is sufficiently drastic to cure the malaise of electromagnetism and of quantum mechanics? As we have shown, retrocausality was built into the very foundations of QED. Yet the T-symmetry of quantum mechanics is routinely squandered in the projection postulate, with its attendant mystique of the measurement process. Might a rigorously atemporal viewpoint lead to a physical picture closer to Einstein's than to Bohr's, and might it be that the infinite self interaction is somehow a mistake induced by our time-bound viewpoint?

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